國立交通大學104學年度碩士班考試入學試題

科目：工程數學(4532)　　　　考試日期：104年2月7日 第3節

系所班別：生物科技學系　　組別：生科系丙組　　第 1 頁，共 7 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

一、Multiple choice questions (2% / question)　60%　請使用答案卡作答

1. We already know that 10% of men and 5% women are color blindness. Now we pick a person randomly from an equal number of men and women in the crowd. What is the probability that this person is female also color blindness?
   (A) $\frac{1}{3}$
   (B) $\frac{2}{3}$
   (C) $\frac{1}{2}$
   (D) $\frac{1}{20}$

2. In a poker game, what is the probability that a five-card hand will contain a triple?
   (A) $\frac{\binom{13}{1}\binom{4}{3}\binom{4}{1}}{\binom{52}{5}}$
   (B) $\frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2}{\binom{52}{5}}$
   (C) $\frac{\binom{13}{1}\binom{4}{3}\binom{4}{2}\binom{44}{1}}{\binom{52}{5}}$
   (D) $\frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}$

3. A lot of N items contain M defectives, and n are selected randomly. What is the probability that n items contain m defectives are selected without replacement?
   (A) $\frac{C_M^m C_{N-M}^{n-m}}{C_N^n}$
   (B) $\frac{C_M^m}{C_N^n}$
   (C) $\frac{C_{N-M}^{n-m}}{C_N^n}$
   (D) $\frac{C_M^m C_{N-M}^{m}}{C_N^n}$

4. Follow the Question 3, what is the probability that n items contain m defectives are selected with replacement?
   (A) $C_n^m\left(\frac{M}{N}\right)^m\left(1-\frac{M}{N}\right)^{n-m}\quad(m=0,1,\cdots,n)$
   (B) $C_n^m\left(\frac{M}{N}\right)^m\quad(m=0,1,\cdots,n)$
   (C) $C_n^m\left(1-\frac{M}{N}\right)^{n-m}\quad(m=0,1,\cdots,n)$
   (D) $C_n^m\left(\frac{M}{N}\right)^{n-m}\quad(m=0,1,\cdots,n)$

5. A pair of dice is tossed 5 times. What is probability of observing at most a total of 5 in any of the five tosses?
   (A) $[1-(\frac{6}{36})^5][1-(\frac{2}{36})^5]$
   (B) $1-(\frac{8}{36})^5$
   (C) $[1-(\frac{6}{36})(\frac{2}{36})]^5$
   (D) $(\frac{10}{36})^5$

6. P(A)=0.8, P(B)=0.6, P(B|A)=0.5, which statement is right?
   (A) $P(A\cap B)=0.4$
   (B) $P(A\cup B)=0.8$
   (C) A and B are independent
   (D) A and B are mutually exclusive

7. A and B run the same experiment independently. The probability of getting a successful experiment was 0.6 and 0.8. What is the probability that one of them was successful?
   (A) 0.4
   (B) 0.42
   (C) 0.44
   (D) 0.48

8. Let X and Y be two independent random variables. Suppose that E[X]=1, E[Y]=2, Var[X]=3, and Var[Y]=4. find $\rho_{X+Y,X-Y}$
   (A) 7
   (B) -1
   (C) $\frac{1}{8}$
   (D) $-\frac{1}{7}$

9. The weekly demand for a certain drink, in thousands of liters, at a chanin of convenience stores is a continuous random variable g(X)=X^2+X-2, where X has the density function. Find the expected value of the weekly demand for the drink.

$$f(x)=\begin{cases}2(x-1) & 1<x<2\\ 0\ , & elsewhere\end{cases}$$

   (A) $\frac{5}{4}$
   (B) $\frac{3}{4}$
   (C) $\frac{3}{2}$
   (D) $\frac{5}{2}$

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10. The joint p.d.f of the random variable X,Y be

$f(x)=\begin{cases}10xy^2, & 0<x<y<1\\ 0, & elsewhere\end{cases}$, find $P(Y>0.5|X=0.25)$

(A) $\frac{3}{5}$

(B) $\frac{3}{4}$

(C) $\frac{4}{7}$

(D) $\frac{8}{9}$

11. Which function is p.d.f

(A) $f_1(x)=\begin{cases}\frac{-5(x-1)}{2} & ,0\le x<1\\ (x-1)^2 & ,1<x<2\\ 0 & ,elsewhere\end{cases}$

(B) $f_2(x)=\begin{cases}3 & ,0\le x\le\frac{1}{2}\\ -1 & ,\frac{1}{2}\le x\le 1\\ 0 & ,elsewhere\end{cases}$

(C) $f_3(x)=\begin{cases}\frac{(x+1)}{2} & ,0\le x\le 1\\ 0 & ,otherwise\end{cases}$

(D) $f_4(x)=\begin{cases}2-4x & ,0\le x\le\frac{1}{2}\\ 4x-2 & ,\frac{1}{2}\le x\le 1\\ 0 & ,elsewhere\end{cases}$

12. Let X and Y have the jpdf f given by

$f(x,y)=ce^{-\frac{x^2-xy+y^2}{2}}, -\infty<x,y<\infty$, find c, fx,fy.

(A) $c=\frac{\sqrt{3}}{2\pi}, fx=\frac{\sqrt{3}}{2\pi}e^{-3x^2/8}, fy=\frac{\sqrt{3}}{2\pi}e^{-3y^2/8}, -\infty<x,y<\infty$

(B) $c=\frac{\sqrt{3}}{\pi}, fx=\frac{1}{2\sqrt{2\pi}}e^{-3x^2/8}, fy=\frac{1}{2\sqrt{2\pi}}e^{-3y^2/8}, -\infty<x,y<\infty$

(C) $c=\frac{\sqrt{3}}{4\pi}, fx=\frac{\sqrt{3}}{2\sqrt{2\pi}}e^{-3x^2/8}, fy=\frac{\sqrt{3}}{2\sqrt{2\pi}}e^{-3y^2/8}, -\infty<x,y<\infty$

(D) $c=\frac{\sqrt{3}}{8\pi}, fx=\frac{\sqrt{3}}{4}e^{-3x^2/8}, fy=\frac{\sqrt{3}}{4}e^{-3y^2/8}, -\infty<x,y<\infty$

13. An urn contains *3* black balls, *2* white balls, and *3* red balls, and *2* balls are chosen without replacement. Let x = the number of black balls and y = the number of white balls. The joint distribution of the numbers of black and white balls in the sample is $\frac{\binom{3}{x}\binom{2}{y}\binom{3}{2-x-y}}{\binom{8}{2}}$, $0\le x\le 2$, find $P(1\le x+y\le 2)$

(A) $\frac{1}{2}$

(B) $\frac{25}{28}$

(C) $\frac{1}{4}$

(D) $\frac{11}{14}$

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14. Let X ~Exp(λ), and Y ~ Exp(λ) be independent. Let Z=x+y, find the fz(z)
   (A) $\lambda e^{-\lambda z}$
   (B) $\lambda^2 z e^{-\lambda z}$
   (C) $\lambda z e^{-\lambda z}$
   (D) $z e^{-\lambda z}$

15. Let x_1 and x_2 are two independent random variables, the variances of x_1 and x_2 are $\sigma_1^2 = 7$ and $\sigma_2^2 = k$ Given that the variance of $y = 5x_1 - 2x_2$ is 21. Find k=?
   (A) 5
   (B) 6
   (C) 7
   (D) 8

16. A box has 3 balls labeled 1,2 and 3 and two balls are drawn without replacement. Let x=the number on the first ball and y=the number on the second ball. Compute E[X+Y] and $E[XY]$
   (A) $E[X+Y] = 2, E[XY] = \frac{1}{3}$
   (B) $E[X+Y] = 3, E[XY] = \frac{5}{2}$
   (C) $E[X+Y] = 4, E[XY] = \frac{11}{3}$
   (D) $E[X+Y] = 6, E[XY] = \frac{9}{4}$

17. Suppose that a measurement has mean μ and variance σ^2=25. Let $\bar{X}$ be the average of n such independent measurements. How large should n be so that $P(|\bar{X} - \mu| < 1) = .957$
   (A) 95
   (B) 96
   (C) 97
   (D) 98

18. Suppose that the error in the reaction temperature, in ℃, for a controlled laboratory experiment is a continuous random variable X having the probability density function.

   $f(x) = \begin{cases} \frac{x^2}{3} & -1 < x < 2 \\ 0, & elsewhere \end{cases}$, Find P(0<X≤1)
   (A) $\frac{1}{3}$
   (B) $\frac{2}{3}$
   (C) $\frac{1}{9}$
   (D) $\frac{2}{9}$

19. The random variables X and Y have joint density function $f(x) = 12xy(1-x)$ $\quad 0 < x, y < 1$ and equal to zero otherwise. Find E[x], Var(y)
   (A) $E[x]=\frac{2}{3}$, $Var(y)=\frac{1}{18}$
   (B) $E[x]=\frac{1}{2}$, $Var(y)=\frac{1}{20}$
   (C) $E[x]=\frac{2}{3}$, $Var(y)=\frac{1}{20}$
   (D) $E[x]=\frac{1}{2}$, $Var(y)=\frac{1}{18}$

20. The joint density function $f(x,y) = \frac{12}{7}(x^2 + xy), 0 \le x \le 1, 0 \le y \le 1$, which description is not correct?
   (A) The marginal density of X is $f_x(x) = \frac{12}{7}(x^2 + \frac{x}{2})$
   (B) The marginal density of Y is $f_y(y) = \frac{12}{7}(\frac{1}{3} + \frac{y}{2})$
   (C) $P(x > y) = \frac{9}{14}$
   (D) $P(y > x) = \frac{5}{14}$

21. Poisson distribution x with λ parameter, find the E[X] and Var[X]
   (A) $E[X]=e^{\lambda}$, $Var[X]=e^{-\lambda}$
   (B) $E[X]=e^{-\lambda}$, $Var[X]=e^{-\lambda}$
   (C) E[X]=λ, Var[X]= -λ
   (D) E[X]=λ, Var[X]= λ

22. The joint density function $f(x,y) = \begin{cases} ke^{-(2x+3y)} & x > 0, y > 0 \\ 0 & elsewhere \end{cases}$
   find k and $P\{0 < X < 1, 0 < Y < 2\}$
   (A) k=6, $P\{0 < X < 1, 0 < Y < 2\} = (1 - e^{-2})(1 - e^{-6})$
   (B) k=5 $P\{0 < X < 1, 0 < Y < 2\} = (1 - e^{-1})(1 - e^{-5})$
   (C) k=3 $P\{0 < X < 1, 0 < Y < 2\} = (1 - e^{-2})(1 - e^{-3})$
   (D) k=2 $P\{0 < X < 1, 0 < Y < 2\} = (1 - e^{-1})(1 - e^{-2})$

23. The random variable K has a normal distribution in a particular interval [0,5]， Find the probability of $f(x)=4x^2 + 4Kx + K + 2 = 0$ with roots。
   (A) 3/5
   (B) 2/5
   (C) 1/5
   (D) 3/4

24. The function $F(x)=\begin{cases}0, & x<0\\ Ax^2, & 0\le x<1\\ 1, & x\ge 1\end{cases}$, find A and $P(0.3\le x\le 0.7)$

(A) $A=1/4$, $P(0.3\le x\le 0.7)=0.1$
(B) $A=1/3$, $P(0.3\le x\le 0.7)=0.2$
(C) $A=1/2$, $P(0.3\le x\le 0.7)=0.3$
(D) $A=1$, $P(0.3\le x\le 0.7)=0.4$

25. Suppose that E[X]=2, E[Y]=1, E[Z]=3, Var[X]=4, Var[Y]=1, Var[Z]=5, Cov(X,Y)=-2, Cov(X,Z)=0, Cov(Y,Z)=2. Let U=3X-2Y+Z, V=X+Y-2Z. find E[U], Var[U] and Cov(U,V)

(A) E[U]=2, Var[U]=61 and Cov(U,V)=9
(B) E[U]=7, Var[U]=15 and Cov(U,V)=17
(C) E[U]=2, Var[U]=15 and Cov(U,V)=-1
(D) E[U]=7, Var[U]=61 and Cov(U,V)=2

26. Suppose $A=A^TA$. which statement is true?

(A) A is idempotent,
(B) $A-I_n$ is idempotent,
(C) all eigenvalues of A are 1,
(D) $A^k=I_n$ for some $k\in\mathbf{N}$.

27. Which statement is false?

(A) Similar matrices have the same eigenvalues and eigenvectors.
(B) If A and C are n x n matrices and C is invertible and v is an eigenvector of A, then $C^{-1}v$ is an eigenvector of $C^{-1}AC$.
(C) Any two n x n diagonal matrices are possible similar.
(D) Two similar n x n matrices represent the same linear transformation of R^n into itself relative to two suitably chosen bases of R^n

28. Identify which of the following statements is true:

(A) Similar matrices always have the same eigenvectors;
(B) Similar matrices always have the same eigenvalues;
(C) Linear operators on infinite-dimensional vector space has n distinct eigenvalues.
(D) Above items are all true.

29. Find the rank of the matrix

$$\begin{bmatrix}1 & 2 & -1 & 3 & 1\\ 0 & 1 & -3 & 2 & 3\\ 2 & 3 & 1 & 4 & -1\\ -1 & 2 & 2 & 2 & -5\\ 3 & 1 & -1 & 2 & 4\end{bmatrix}$$

(A) 1
(B) 2
(C) 3
(D) 4

30. Find the rank of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(A) 1
(B) 2
(C) 3
(D) 4

二、Calculation and Proof Questions 40% 非選擇題請用答案卷作答

1. (10%) $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$, $P(A)=3A^5+2A^3+I$, please calculate the eigenvalues of P(A).

2. (15%) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ Please calculate the following items.
   (1) Eigenvalues of A (5%)
   (2) Eigenvectors of A (5%)
   (3) Eigenvalues of A^2 (5%)

3. (15%) Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$
   (a) Please prove that A, B, and C are similar, that is, they belong to the same family which can be characterized by a Jordan form. (10%)
   (b) Derive their common Jordan form (an upper triangular matrix). (5%)