

國立交通大學 104 學年度碩士班考試入學試題

科目：微積分(4531)

第 4544 微積分同試題

考試日期：104 年 2 月 7 日 第 3 節

系所班別：生物科技學系

組別：生科系丙組、分醫所

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【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. To calculate

(a) (3%) Given $p > 1$, $\int_1^{\infty} \frac{1}{x^p} dx = ?$

(b) (3%) $\lim_{n \rightarrow \infty} \frac{2n^5}{1^2 + 2^2 + \dots + n^2} = ?$

(c) (4%) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 9}}{3x + 6} = ?$

2. To evaluate

(a) (4%) $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^5}$

(b) (4%) $\lim_{x \rightarrow 0} \frac{(e^{2x} - 1) \ln(1 + x^5)}{(1 - \cos 3x)^2}$

3. To determine whether the given series converges or diverges by any appropriate test.

(a) (4%) $\sum_{n=1}^{\infty} \left| \sin \frac{1}{n^2} \right|$

(b) (4%) $\sum_{n=1}^{\infty} \frac{1 + n^{4/5}}{2 + n^{5/5}}$

(c) (4%) $\sum_{n=1}^{\infty} \frac{n!}{n^2 e^n}$

4. Let

$$f(x) = \sum_{k=0}^{\infty} \frac{2^{2k} k!}{(2k+1)!} x^{2k+1} = x + \frac{2}{3} x^3 + \frac{4}{3 \times 5} x^5 + \frac{8}{3 \times 5 \times 7} x^7 + \dots$$

(a) (5%) Find the radius of convergence of this power series.

(b) (5%) Show that $f'(x) = 1 + 2xf(x)$.

(c) (5%) What is $\frac{d}{dx} (e^{-x^2} f(x))$?

(d) (5%) Express $f(x)$ in terms of an integral.

5. (14%) At the critical point, gases obey the following equations:

$$\left(\frac{\partial P}{\partial V_m} \right)_T = 0 \quad \text{and} \quad \left(\frac{\partial^2 P}{\partial V_m^2} \right)_T = 0$$

where P is pressure, T is temperature and V_m is molar volume. If a certain gas behaviors like van der Waals gas, $P = \frac{RT}{(V_m - b)} - \frac{a}{V_m^2}$, show that the relation between critical point constants (V_{mC} ; T_C ; P_C) and van der Waals coefficients (a and b) as following:

$$V_{mC} = 3b; T_C = \frac{8a}{27Rb}; P_C = \frac{a}{27b^2}$$

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6. (12%) The change of the enthalpy H could be represented as following:

$$dH = \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP$$

where $\left(\frac{\partial H}{\partial P}\right)_T$ indicates how enthalpy changes with pressure under constant temperature and it obeys the equation shown below:

$$\left(\frac{\partial H}{\partial P}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_P$$

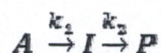
where P is pressure, T is temperature and V is volume.

- (a) Assume argon is an ideal gas, show that $\left(\frac{\partial H}{\partial P}\right)_T = 0$. (Ideal gas law: $PV=nRT$, where R is gas constant)
- (b) Calculate ΔH for the *isothermal* reversible expansion of 1 mole argon for an initial volume of V_1 to final volume of V_2 at *constant temperature* of T .

7. (24%) The restriction enzyme EcoRI catalyses the cleavage of DNA and bring about the sequence of reactions:

Supercoiled DNA \rightarrow *open circle DNA* \rightarrow *linear DNA*

To illustrate the kinds of consideration involved, we suppose the a reaction takes place in two steps. First an intermediate I (the open circle DNA) is formed from the reactant A (the supercoiled DNA) in a first-order reaction with reaction k_1 . Then I decays in a first-order reaction with rate k_2 to form the product P (linear DNA):



The initial conditions for this reaction are $[A]_0 = A_0$ and $[I]_0 = [P]_0 = 0$. Also at all time, the relationship between the concentrations of all species is $[A]_t + [I]_t + [P]_t = A_0$.

(a) All the steps in the reaction scheme are elementary reactions. Express $\frac{d[A]}{dt}$, $\frac{d[I]}{dt}$, and $\frac{d[P]}{dt}$ in terms of reaction constants (k_1 or k_2) and the concentration of all related species ($[A]$ or $[I]$).

(b) Derive that $[A]_t = A_0 e^{-k_1 t}$

(c) From (a) and (b), then derive that :

$$[I]_t = \frac{k_1}{k_2 - k_1} A_0 (e^{-k_1 t} - e^{-k_2 t})$$

(d) Derive $[P]_t$ in term of A_0 , k_1 , and k_2 , using a relationship of $[A]_t + [I]_t + [P]_t = A_0$.

(e) In the case of $k_1 \gg k_2$, all of the A initially present is rapidly converted into I , which is then slowly used up to

form C . Show that $[C]_t = [1 - \exp(-k_2 t)]A_0$.

(f) From (c), when would the intermediate reaches to its maximum concentration.