國立交通大學104學年度碩士班考試入學試題

科目：微積分(4531) 與4544微積分同試題 考試日期：104年2月7日 第3節

系所班別：生物科技學系分醫所 組別：生科系丙組、分醫所 第1頁，共2頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. To calculate
   (a) (3%) Given $p > 1$, $\int_1^\infty \frac{1}{x^t}\,dx = ?$
   (b) (3%) $\lim\limits_{n\to\infty} \frac{2n^3}{1^2+2^2+\cdots+n^2} = ?$
   (c) (4%) $\lim\limits_{x\to-\infty} \frac{\sqrt{x^2+9}}{3x+6} = ?$

2. To evaluate
   (a) (4%) $\lim\limits_{x\to 0} \frac{x-\sin x}{x^3}$
   (b) (4%) $\lim\limits_{x\to 0} \frac{(e^{2x}-1)\ln(1+x^3)}{(1-\cos 3x)^2}$

3. To determine whether the given series converges or diverges by any appropriate test.
   (a) (4%) $\sum_{n=1}^{\infty} \left|\sin \frac{1}{n^2}\right|$
   (b) (4%) $\sum_{n=1}^{\infty} \frac{1+n^{4/3}}{2+n^{5/3}}$
   (c) (4%) $\sum_{n=1}^{\infty} \frac{n!}{n^2 e^n}$

4. Let

$$f(x) = \sum_{k=0}^{\infty} \frac{2^{2k}\,k!}{(2k+1)!} x^{2k+1} = x + \frac{2}{3}x^3 + \frac{4}{3\times 5}x^5 + \frac{8}{3\times 5\times 7}x^7 + \cdots$$

   (a) (5%) Find the radius of convergence of this power series.
   (b) (5%) Show that $f'(x) = 1 + 2xf(x)$.
   (c) (5%) What is $\frac{d}{dx}\left(e^{-x^2} f(x)\right)$?
   (d) (5%) Express $f(x)$ in terms of an integral.

5. (14%) At the critical point, gases obey the following equations:

$$\left(\frac{\partial P}{\partial V_m}\right)_T = 0 \qquad and \qquad \left(\frac{\partial^2 P}{\partial {V_m}^2}\right)_T = 0$$

where P is pressure, T is temperature and V_m is molar volume. If a certain gas behaviors like van der Waals gas, $P = \frac{RT}{(V_m-b)} - \frac{a}{{V_m}^2}$, show that the relation between critical point constants (V_{m_C}; T_C; P_C) and van der Waals coefficients (a and b) as following:

$$V_{m_C} = 3b;\ T_C = \frac{8a}{27Rb};\ P_C = \frac{a}{27b^2}$$

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6. (12%) The change of the <u>*enthalpy H*</u> could be represented as following:

$$dH = \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP$$

where $\left(\frac{\partial H}{\partial P}\right)_T$ indicates how enthalpy changes with pressure under constant temperature and it obeys the equation shown below:

$$\left(\frac{\partial H}{\partial P}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_P$$

where P is pressure, T is temperature and V is volume.

(a) Assume argon is an ideal gas, show that $\left(\frac{\partial H}{\partial P}\right)_T = 0$. (Ideal gas law: PV=nRT, where R is gas constnat)

(b) Calculate ΔH for the *isothermal* reversible expansion of 1 mole argon for an initial volume of V_1 to final volume of V_2 at *constant temperature* of T.

7. (24%) The restriction enzyme EcoRI catalyses the cleavage of DNA and bring about the sequence of reactions:

$Supercoiled\ DNA \longrightarrow open\ circle\ DNA \longrightarrow linear\ DNA$

To illustrate the kinds of consideration involved, we suppose the a reaction takes place in two steps. First an intermediate $\boldsymbol{I}$ (the open circle DNA) is formed from the reactant $\boldsymbol{A}$ (the supercolied DNA) in a first-order reaction with reaction k_1. Then I decays in a first-order reaction with rate k_2 to form the product $\boldsymbol{P}$ (linear DNA):

$$\boldsymbol{A \overset{k_1}{\rightarrow} I \overset{k_2}{\rightarrow} P}$$

The initial conditions for this reaction are $[A]_0 = A_0$ and $[I]_0 = [P]_0 = 0$. Also at all time, the relationship between the concentrations of all species is $[A]_t + [I]_t + [P]_t = A_0$.

(a) All the steps in the reaction scheme are elementary reactions. Express $\frac{d[A]}{dt}$, $\frac{d[I]}{dt}$, and $\frac{d[P]}{dt}$ in terms of reaction constants (k_1 or k_2) and the concentration of all related species ([A] or [I]).

(b) Derive that $[A]_t = A_0 e^{-k_1 t}$

(c) From (a) and (b), then derive that :

$$[I]_t = \frac{k_1}{k_2 - k_1} A_0 (e^{-k_1 t} - e^{-k_2 t})$$

(d) Derive $[P]_t$ in term of A_0, k_1, and k_2, using a relationship of $[A]_t + [I]_t + [P]_t = A_0$.

(e) In the case of $k_1 \gg k_2$, all of the A initially present is rapidly converted into I, which is then slowly used up to
form C. Show that $[C]_t = [1 - \exp(-k_2 t)]A_0$.

(f) From (c), when would the intermediate reaches to its maximum concentration.