

國立彰化師範大學 104 學年度 碩士班 招生考試試題

系所： 統計資訊研究所

科目： 統計

☆☆請在答案紙上作答☆☆

共 5 頁，第 1 頁

1. (25%)

- (a) Show that if two normally distributed random variables are uncorrelated, they are independent.
- (b) Let X_1, \dots, X_n be iid normally distributed random variables. Show that their sample mean and sample variance are independent.

2. (25%)

Let X_1, X_2, \dots, X_m i.i.d $Exp(\lambda)$

- (a) Use Maximum Likelihood Estimation to estimate λ .
- (b) Discuss if the MLE $\hat{\lambda}$ is unbiased. Why?

3. (30%)

Consider the regression model:

$$Y = \beta_0 \mathbf{1} + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon = \mathbf{X}\boldsymbol{\beta} + \varepsilon,$$

where Y is the vector of responses, $\mathbf{1}$, X_1 , X_2 and X_3 are the corresponding vectors of independent variables for the regression coefficients β_0 , β_1 , β_2 and β_3 , $\mathbf{X} = (\mathbf{1}, X_1, X_2, X_3)$, and $\mathbf{1}$ is a vector of 1s. Here ε follows a multivariate normal distribution, $MN(\mathbf{0}, \sigma^2 \mathbf{I})$ where \mathbf{I} is an identity matrix.

A sample of 25 observations was obtained, with the results summarized in the following calculations.

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.004 & 0 \\ 0 & 0 & 0 & 0.006 \end{bmatrix}, \quad \mathbf{X}'\mathbf{Y} = \begin{bmatrix} 1000 \\ 300 \\ 500 \\ 100 \end{bmatrix}, \quad \sum_{i=1}^{25} y_i^2 = 22162 \quad \text{and} \quad 25\bar{y}^2 = 20474.$$

- (a) Please calculate the least squares estimates of the parameters β_0 , β_1 , β_2 and β_3 . (4%)
- (b) Please fill in the blank (1)~(5) in the following ANOVA table. (8%)

Source of variation	SS	DF
Regression	(1) (2%)	(4) (1%)
Error	(2) (2%)	(5) (1%)
Total	(3) (2%)	24

- (c) What are the coefficient of multiple determination and the adjusted coefficient of multiple determination for the regression model. (4%)
- (d) Do the three explanatory variables X_1 , X_2 and X_3 (consider together) have a statistically significant effect on the response Y ? Please specify the null and alternative hypotheses, the test used, and your conclusion using a 5% level of significance. (6%)
- (e) Please find the estimated standard errors for the estimates of β_0 , β_1 , β_2 and β_3 . (4%)

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共 5 頁，第 2 頁

(f) To test $H_0: \beta_1 = 0$. Please specify the test used, distribution of the testing statistic and your conclusion. Use $\alpha = 0.05$. (4%)

4. (20%)

The mean body weight for adults 20-59 years old is believed to have a normal distribution. A sample of 200 adults gives the data shown as follows. Do these data support the belief that the mean body weight has a normal distribution? Assume that the mean and standard deviation of this hypothetical normal distribution are given by the sample mean, $\bar{X} = 75$, and sample standard deviation, $S = 10$, respectively. Use a 5% level of significance. Please specify the null and alternative hypotheses, the test used, the distribution of the testing statistic and your conclusion.

Group(Kg)	Observed frequency
< 50	2
$\geq 50, < 60$	13
$\geq 60, < 70$	45
$\geq 70, < 80$	75
$\geq 80, < 90$	50
≥ 90	15
Total	200

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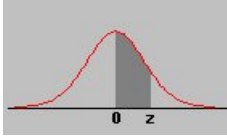
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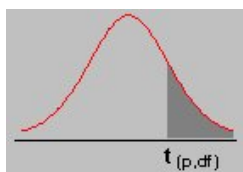
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附表：



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990



df\Pr	0.25	0.1	0.05	0.025	0.01	0.005
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707
7	0.711	1.415	1.895	2.365	2.998	3.499
8	0.706	1.397	1.860	2.306	2.896	3.355
9	0.703	1.383	1.833	2.262	2.821	3.250
10	0.700	1.372	1.812	2.228	2.764	3.169
11	0.697	1.363	1.796	2.201	2.718	3.106
12	0.695	1.356	1.782	2.179	2.681	3.055
13	0.694	1.350	1.771	2.160	2.650	3.012
14	0.692	1.345	1.761	2.145	2.624	2.977
15	0.691	1.341	1.753	2.131	2.602	2.947

df\Pr	0.25	0.1	0.05	0.025	0.01	0.005
16	0.690	1.337	1.746	2.120	2.583	2.921
17	0.689	1.333	1.740	2.110	2.567	2.898
18	0.688	1.330	1.734	2.101	2.552	2.878
19	0.688	1.328	1.729	2.093	2.539	2.861
20	0.687	1.325	1.725	2.086	2.528	2.845
21	0.686	1.323	1.721	2.080	2.518	2.831
22	0.686	1.321	1.717	2.074	2.508	2.819
23	0.685	1.319	1.714	2.069	2.500	2.807
24	0.685	1.318	1.711	2.064	2.492	2.797
25	0.684	1.316	1.708	2.060	2.485	2.787
26	0.684	1.315	1.706	2.056	2.479	2.779
27	0.684	1.314	1.703	2.052	2.473	2.771
28	0.683	1.313	1.701	2.048	2.467	2.763
29	0.683	1.311	1.699	2.045	2.462	2.756
30	0.683	1.310	1.697	2.042	2.457	2.750
inf	0.674	1.282	1.645	1.960	2.326	2.576

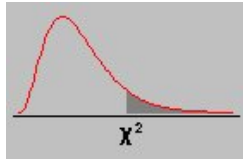
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df\Pr	0.1	0.05	0.025	0.01	0.005	0.001
1	2.706	3.841	5.024	6.635	7.879	10.828
2	4.605	5.991	7.378	9.21	10.597	13.816
3	6.251	7.815	9.348	11.345	12.838	16.266
4	7.779	9.488	11.143	13.277	14.86	18.467
5	9.236	11.07	12.833	15.086	16.75	20.515
6	10.645	12.592	14.449	16.812	18.548	22.458
7	12.017	14.067	16.013	18.475	20.278	24.322
8	13.362	15.507	17.535	20.09	21.955	26.124
9	14.684	16.919	19.023	21.666	23.589	27.877
10	15.987	18.307	20.483	23.209	25.188	29.588
11	17.275	19.675	21.92	24.725	26.757	31.264
12	18.549	21.026	23.337	26.217	28.3	32.909
13	19.812	22.362	24.736	27.688	29.819	34.528
14	21.064	23.685	26.119	29.141	31.319	36.123
15	22.307	24.996	27.488	30.578	32.801	37.697
16	23.542	26.296	28.845	32	34.267	39.252
17	24.769	27.587	30.191	33.409	35.718	40.79
18	25.989	28.869	31.526	34.805	37.156	42.312
19	27.204	30.144	32.852	36.191	38.582	43.82
20	28.412	31.41	34.17	37.566	39.997	45.315
21	29.615	32.671	35.479	38.932	41.401	46.797
22	30.813	33.924	36.781	40.289	42.796	48.268
23	32.007	35.172	38.076	41.638	44.181	49.728
24	33.196	36.415	39.364	42.98	45.559	51.179
25	34.382	37.652	40.646	44.314	46.928	52.62

df\Pr	0.1	0.05	0.025	0.01	0.005	0.001
26	35.563	38.885	41.923	45.642	48.29	54.052
27	36.741	40.113	43.195	46.963	49.645	55.476
28	37.916	41.337	44.461	48.278	50.993	56.892
29	39.087	42.557	45.722	49.588	52.336	58.301
30	40.256	43.773	46.979	50.892	53.672	59.703
31	41.422	44.985	48.232	52.191	55.003	61.098
32	42.585	46.194	49.48	53.486	56.328	62.487
33	43.745	47.4	50.725	54.776	57.648	63.87
34	44.903	48.602	51.966	56.061	58.964	65.247
35	46.059	49.802	53.203	57.342	60.275	66.619
36	47.212	50.998	54.437	58.619	61.581	67.985
37	48.363	52.192	55.668	59.893	62.883	69.346
38	49.513	53.384	56.896	61.162	64.181	70.703
39	50.66	54.572	58.12	62.428	65.476	72.055
40	51.805	55.758	59.342	63.691	66.766	73.402
41	52.949	56.942	60.561	64.95	68.053	74.745
42	54.09	58.124	61.777	66.206	69.336	76.084
43	55.23	59.304	62.99	67.459	70.616	77.419
44	56.369	60.481	64.201	68.71	71.893	78.75
45	57.505	61.656	65.41	69.957	73.166	80.077
46	58.641	62.83	66.617	71.201	74.437	81.4
47	59.774	64.001	67.821	72.443	75.704	82.72
48	60.907	65.171	69.023	73.683	76.969	84.037
49	62.038	66.339	70.222	74.919	78.231	85.351
50	63.167	67.505	71.42	76.154	79.49	86.661

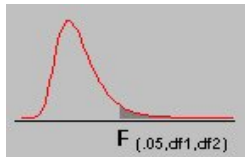
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df2/df1	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18