國立嘉義大學 104 學年度

應用數學系碩士班（甲組）招生考試試題

科目：微積分

說明：本考試試題為計算、證明題，每題 10 分，請標明題號，同時將過程作答在「答案卷」上。

1. Evaluate the limits.　(a)　$\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{x}$ 　(b)　$\lim\limits_{x\to\infty}\dfrac{\ln x}{x}$

2. Find each integral.　(a)　$\int xe^x dx$ 　(b)　$\int \dfrac{4x}{x^2+9}dx$

3. Find the absolute maximum and minimum values of　$f(x)=\sqrt{4-x^2}$, $\;-2\le x\le 1$.

4. Show that there is a real root of the equation　$x^3+x-1=0$　between 0 and 1.

5. Find a power series for $f(x)=\ln x$, centered at 1, and determine the interval of convergence.

6. Find the derivative of the following functions

   (a)　$f(x)=\dfrac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$.　　(b)　$g(x)=arc\sec e^{3x}$.

7. Show that　$f(x)=\begin{cases} x^2+2x & \text{if } x \text{ is rational,} \\ 2x & \text{if } x \text{ is irrational.} \end{cases}$　is continuous, differentiable at　$x=0$

   and find the value of　$f'(0)$.

8. Find the area between　$y=2\sec^2 x$　and　$y=\sin x$　from　$x=0$　to　$x=\dfrac{\pi}{4}$.

9. Evaluate the following limits

   (a)　$\lim\limits_{(x,y)\to(0,0)}\dfrac{2x^2 y}{x^2+y^2}$.　　(b)　$\lim\limits_{(x,y)\to(1,1)}\dfrac{\sqrt{y}-\sqrt{x+1}}{y-x-1}$.

10. Evaluate the double integral　$\iint_{\Omega} y\,e^x\,dxdy$　on the closed set

   $\Omega=\left\{(x,\,y)\in\Re^2 \,\middle|\, 0\le y\le 1,\,0\le x\le y^2\right\}$.