

科目：工程數學 適用：應光系

編號：501

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. (10%) Solve $y^{(4)} + 2y'' + y = 0, y = y(x)$

2. (10%) Solve $y^{(3)} + y'' = e^x \cos x, y = y(x)$

3. (15%) Solve the general solution first and then use "variation of parameters" to obtain the particular solution:

$$x^2 y'' - 3xy' + 3y = 2x^2 e^x \quad y = y(x)$$

4. (20%) Solve the following ODE by power series method. You ONLY need to show the first THREE terms of each of your solutions.

$$y'' + (\cos x)y = 0, y = y(x) \text{ (Hint: } \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \dots \text{)}$$

5. The Laplace Transform can be expressed in this way $F(s) = L(f) = \int_0^\infty e^{-st} f(t) dt$

(5%) Prove that $L(f') = sL(f) - f(0)$

(5%) Prove that $L(f'') = s^2 L(f) - sf(0) - f'(0)$

(20%) Please use the relation above and the given table below to solve the following differential equation

For $y = y(t)$

$$y'' + 9y = e^t, y(0) = 0, y'(0) = 0$$

$f(t)$	$L(f)$
1	$1/s$
e^{at}	$1/(s - a)$
$\cos(\omega t)$	$s/(s^2 + \omega^2)$
$\sin(\omega t)$	$\omega/(s^2 + \omega^2)$

6. (15%) Expand $f(x) = e^{-x}, -\pi < x < \pi$, in a complex Fourier series.

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