

科目：工程數學三(機率)

適用：電機系(通訊工程)

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. (20%)

- (a) (8%) Compute the probability generating function $\Phi(t)$ of a geometric density function with parameter p . ($\text{Prob}(X = k) = (1 - p)^{k-1}p, k = 1, 2, 3, \dots$)
- (b) (12%) Use the result in (a) to find the mean and variance of the density.

2. (20%) Suppose n balls are distributed at random into r boxes. Let $X_i = 1$ if box i is empty and let $X_i = 0$ otherwise.

- (a) (5%) Compute $E(X_i)$.
- (b) (7%) For $i \neq j$, compute $E(X_i X_j)$.
- (c) (8%) Let $S_r = X_1 + X_2 + \dots + X_r$. Find the mean and variance of S_r .

3. (25%) Let X have the normal density $\mathcal{N}(0, \sigma^2)$, indicating that X has a zero mean and a variance of σ^2 .

- (a) (5%) Find the probability density of $Y = 2X + 3$.
- (b) (8%) Find the probability density of $Y = X^2$.
- (c) (12%) Find the probability density of $Y = \exp(X)$.

4. (25%) Let X and Y be independent random variables having respective normal densities $\mathcal{N}(1, 4)$ and $\mathcal{N}(-2, 9)$.

- (a) (5%) Find the probability density of $Z_1 = X - 2Y$.
- (b) (8%) Find the mean and variance of $Z_2 = -X + Y$.
- (c) (12%) Find the covariance of Z_1 and Z_2 defined in (a) and (b).

5. (10%) Let X_1, X_2, \dots, X_n be independent random variables having a common normal density with mean μ and variance σ^2 . Find two constants α and β such that the following random variable

$$Y = \frac{(X_1 + X_2 + \dots + X_n) - \alpha}{\beta}$$

has the same probability density as X_1 .