

科目：線性代數 適用：電機系(系統組)、電機系(通訊工程)

考生注意：
 1. 依次序作答，只要標明題號，不必抄題。
 2. 答案必須寫在答案卷上，否則不予計分。
 3. 限用藍、黑色筆作答；試題須隨卷繳回。

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編號：461.471

1. Determine which of the following are subspaces of \mathbb{R}^3 . (20%)

- (a) All vectors of the form $(0, c, 0)$, where c is real-valued. (5%)
- (b) All vectors of the form $(1, c, 0)$, where c is real-valued. (5%)
- (c) All vectors of the form (a, b, c) , where $b = a + c$ and a, b and c are real-valued. (5%)
- (d) All vectors of the form $(c, 0, -c)$, where c is real-valued (5%)

2. Which of the following sets of vectors are a basis of \mathbb{R}^4 ? (15%)

- (a) $(1, 2, -2, 1), (3, 6, -5, 3), (4, -2, 4, 1)$. (5%)
- (b) $(1, 0, -6, 3), (0, 1, 3, 0), (0, 2, 7, 0), (0, 2, 0, 1)$. (5%)
- (c) $(0, 0, 0, 0), (1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)$. (5%)

3. In each part, compute the stated power of $A = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$. (10%)

- (a) A^{10} . (5%)
- (b) A^∞ . (5%)

4. Let \mathbb{R}^3 have the inner product $\langle u, v \rangle = u_1v_1 + 2u_2v_2 + 3u_3v_3$. Use the Gram-Schmidt process to transform $x_1 = (1, 1, 1), x_2 = (1, 1, 0), x_3 = (1, 0, 0)$ into an orthonormal basis. (15%)

5. (10%)

- (a) If A has three orthogonal columns each of length 4, what is $A^T A$? (5%)
- (b) If A has four orthogonal columns of length 1, 2, 5 and 8, respectively, what is $A^T A$? (5%)

6. Find the projection of b onto the column space of A . (15%)

(a) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$. (7%)

(b) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$. (8%)

7. True or false. (15%)

(a) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 0 \end{bmatrix}$. Then the column space of A is \mathbb{R}^3 . (3%)

- (b) If the columns of a square matrix are independent, so are the rows. (3%)
- (c) A diagonalizable matrix is always invertible. (3%)
- (d) An invertible matrix is always diagonalizable. (3%)
- (e) Two similar matrices share the same eigenvalues. (3%)