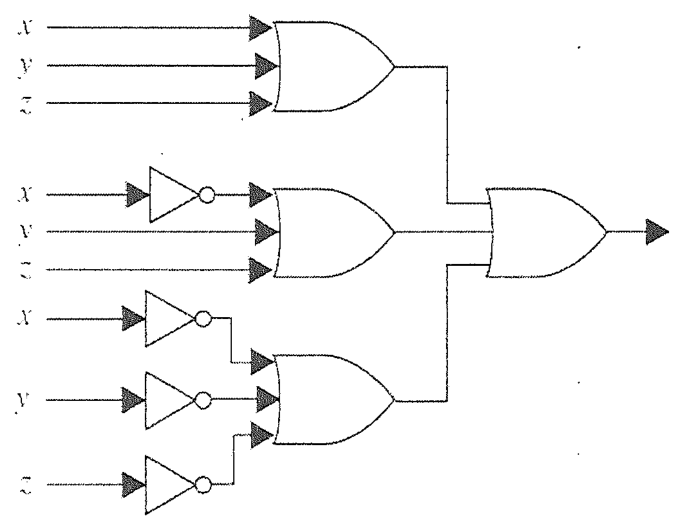
國立臺北大學104學年度碩士班一般入學考試試題

系（所）組別：資訊工程學系

科　　　目：線性代數與離散數學　　　　第1頁 共1頁

□可 ☑不可使用計算機

1. (10%) Find 3^{2003} mod 99.
2. (10%) A Cantor expansion is a sum of the form $a_n n! + a_{n-1}(n-1)! + \cdots + a_2 2! + a_1 1!$, where a_i is an integer with $0 \le a_i \le i$ for i = 1, 2,…,n. Find the Cantor expansion of the integer 1000000.
3. (10%) Fibonacci numbers, $f_0, f_1, f_2, \ldots$, are defined by the equations $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for n = 2, 3, 4,…Prove that $f_{n+1}f_{n-1} - (f_n)^2 = (-1)^n$ when n is a positive integer.
4. (10%) The set of all neighbors of a vertex v of G = (V ,E), denoted by N(v), is called the neighborhood of *v*. Define deg(v)=|N(v)|. Let G = (V ,E) be an undirected graph with m edges. Prove that $2m = \sum_{v \in V} \deg(v)$.
5. (10%) Find the output of the following circuit.



6. (20%) Show the details of finding the complete solution to $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ with the following $\mathbf{A}$, $\boldsymbol{x}$, and $\boldsymbol{b}$.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 8 & 12 \\ 3 & 6 & 7 & 13 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \text{ and } \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

7. (15%) Find the eigenvalues and eigenvectors of the following matrix A.

$$\mathbf{A} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}.$$

8. (15%) Show the details of testing the following matrix A for positive definiteness.

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

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