科目：線性代數 適用：電機系(系統組)、電機系(通訊工程)

編號：461. 471

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

1. Determine which of the following are subspaces of $\mathbb{R}^3$. (20%)
   (a) All vectors of the form $(0,\ c,\ 0)$, where c is real-valued. (5%)
   (b) All vectors of the form $(1,\ c,\ 0)$, where c is real-valued. (5%)
   (c) All vectors of the form $(a,\ b,\ c)$, where $b = a + c$ and a, b and c are real-valued. (5%)
   (d) All vectors of the form $(c,\ 0,\ -c)$, where c is real-valued (5%)
2. Which of the following sets of vectors are a basis of $\mathbb{R}^4$? (15%)
   (a) $(1,\ 2,\ -2,\ 1), (3,\ 6,\ -5,\ 3), (4,\ -2,\ 4,\ 1)$. (5%)
   (b) $(1,\ 0,\ -6,\ 3), (0,\ 1,\ 3,\ 0), (0,\ 2,\ 7,\ 0), (0,\ 2,\ 0,\ 1)$. (5%)
   (c) $(0,\ 0,\ 0,\ 0), (1,\ 0,\ 0,\ 0), (0,\ 1,\ 0,\ 0), (0,\ 0,\ 1,\ 0), (0,\ 0,\ 0,\ 1)$. (5%)
3. In each part, compute the stated power of $A = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$. (10%)
   (a) A^{10}. (5%)
   (b) A^{∞}. (5%)
4. Let $\mathbb{R}^3$ have the inner product $\langle \mathbf{u}, \mathbf{v} \rangle = u_1v_1 + 2u_2v_2 + 3u_3v_3$. Use the Gram-Schmidt process to transform $\mathbf{x}_1 = (1,\ 1,\ 1)$, $\mathbf{x}_2 = (1,\ 1,\ 0)$, $\mathbf{x}_3 = (1,\ 0,\ 0)$ into an orthonormal basis. (15%)
5. (10%)
   (a) If A has three orthogonal columns each of length 4, what is A^TA? (5%)
   (b) If A has four orthogonal columns of length 1, 2, 5 and 8, respectively, what is A^TA? (5%)
6. Find the projection of $\mathbf{b}$ onto the column space of A. (15%)
   (a) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$. (7%)
   (b) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$. (8%)
7. True or false. (15%)
   (a) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 0 \end{bmatrix}$. Then the column space of A is $\mathbb{R}^3$. (3%)
   (b) If the columns of a square matrix are independent, so are the rows. (3%)
   (c) A diagonalizable matrix is always invertible. (3%)
   (d) An invertible matrix is always diagonalizable. (3%)
   (e) Two similar matrices share the same eigenvalues. (3%)