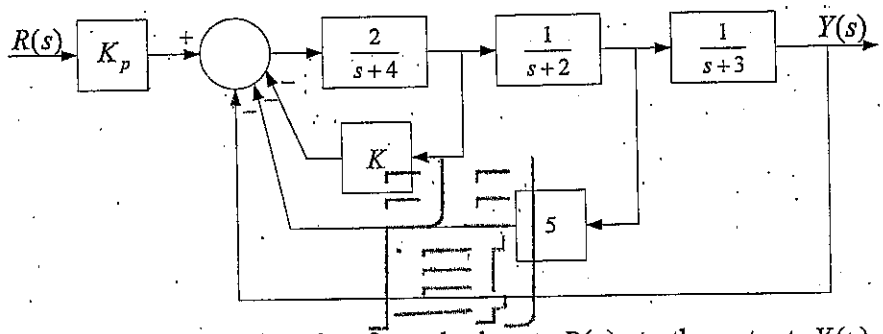
1. (20%) A linear time-invariant single-input-single-output physical system is described by the following ordinary differential equation with the output $y(t)$, the input $u(t)$ and the constant coefficients a_i and b_i, $i=1,2,3$:

$$3\dddot{y}(t)+a_3\ddot{y}(t)+a_2\dot{y}(t)+a_1y(t)=b_1\ddot{u}(t)+b_2\dot{u}(t)+b_3u(t).$$

(a) Determine the input-output relationship of this system with both transfer function and state variable models. (10%)

(b) Find the conditions on a_i and b_i, $i=1,2,3$ for the stability of this system. (10%)

2. (40%) The feedback structure of an automobile suspension system is shown as follows:



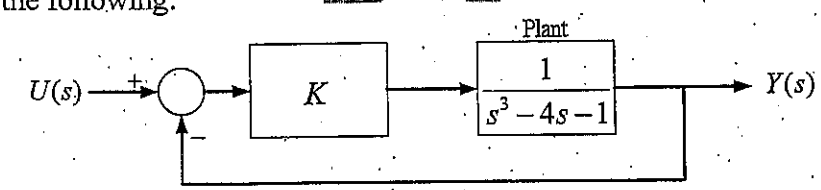
(a) Determine the transfer function from the input $R(s)$ to the output $Y(s)$. (10%)

(b) Select the feedback gain K so that the characteristic equation has three integer roots lying in the interval $-6<s<-3$. What are those roots? (10%)

(c) With the choice of the gain K in Part **(b)**, select K_p so that the steady-state error $\lim_{t\to\infty}e(t)$ (where $e(t)=r(t)-y(t)$) for a unit step input is equal to zero. (10%)

(d) From Parts **(b)** and **(c)**, find the output response $y(t)$ to a unit step input. (10%)

3. (40%) A unity feedback control system with the output $y(t)$ and the input $u(t)$ is shown in the following:



(a) Determine the state variable model for this system with the given state assignment $x(t)=[x_1(t)\quad x_2(t)\quad x_3(t)]^T=[y(t)\quad \dot{y}(t)\quad \ddot{y}(t)]^T$. (10%)

(b) If $K=1$, find the equivalence transformation $\bar{x}(t)=Px(t)$ that transforms the given state equations into the diagonal canonical form if possible. (10%)

(c) Find the zero-input response of this system if $K=1$ and $x(0)=[1\quad 0\quad 1]^T$. (10%)

(d) If the control input $u(t)=-[k_1\quad k_2\quad k_3]\bar{x}(t)$, where $\bar{x}(t)$ is obtained from Part **(b)**, is used for state feedback design, select the values of k_1, k_2 and k_3 so that all the poles of the resulting closed-loop system are repeated at $s=-2$. (10%)