

國立臺北大學 104 學年度碩士班一般入學考試試題

系(所)組別：統計學系
科 目：基礎數學

第 1 頁 共 1 頁
可 不可使用計算機

一、微積分計算

注意事項

- 每題計分標於題目之後，請依序作答
- 請詳述計算過程，只有填寫答案無計算或證明過程不予計分

1. Find $\lim_{x \rightarrow 0} \frac{\sin 2x(1-\cos 2x)}{x^{\frac{8}{3}} \sin(x^{\frac{1}{3}})} = ?$ (8%)
2. Find derivative df/dx for $f(x) = 10^x \cdot x^{10} + x^2 \log_{10}(x^2 + 1)^5 + 7^x \ln(x - 1)$. (8%)
3. Find the integral $\int \sqrt{\frac{1-x}{1+x}} dx = ?$ (8%)
4. Find the integral $\int \frac{\sin x}{\cos^2 x - 6 \cos x + 8} dx = ?$ (8%)
5. Find the area bounded by the curves $x^2 + y^2 \leq 4$ and $y > 4x^2$. (8%)
6. Find the double integral $\iint_A x^2 y \, dx dy$, where $A = \{0 \leq x^2 + y^2 \leq 4, y > 0\}$. (10%)

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1. Please orthogonally diagonalize the matrix $A = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$ (That is, find an orthogonal matrix P and a diagonal matrix D such that $D = P^T A P$). (14%)
2. Let $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ be a basis for P_2 and let $\bar{w}_1 = -\bar{v}_1 + 2\bar{v}_2 + 2\bar{v}_3$, $\bar{w}_2 = 2\bar{v}_1 - \bar{v}_2 + 2\bar{v}_3$, $\bar{w}_3 = 2\bar{v}_1 + 2\bar{v}_2 - \bar{v}_3$. Please show that $\{\bar{w}_1, \bar{w}_2, \bar{w}_3\}$ is also a basis for P_2 . (12%)
3. Let $L: V \rightarrow W$ be a linear transformation mapping from vector space V to vector space W. Please show that the kernel of L is a subspace of V. (5%)
4. Let A be an nxn matrices with $A^n = O$ for some positive integer n. Please show that A is singular. (5%)
5. Let $A = \begin{pmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{pmatrix}$.
 - (a) Please find $\det(A)$. (10%)
 - (b) Is A nonsingular? Why? If so, please find $\det(A^{-1})$. (4%)

試題隨卷繳交