統計學、微積分皆出題於同張試題內，請務必擇一科目作答，若兩科均作答，則以統計學的成績計算。

※微積分(100%)

1. (20%) Evaluate the following limit.

    A. $\lim_{x \to \infty} x^{\frac{2}{x}}$

    B. $\lim_{t \to \infty} \dfrac{2(e^{-2t} + t^2)}{e^{-2t} - t^2}$

    C. $\lim_{t \to 5} \dfrac{t^2 - 3t - 10}{t - 5}$

    D. $\lim_{x \to \infty} \left(x^2 \sin \dfrac{2}{x^2} \right)$

2. (20%) Find dy/dx.

    A. $y = \left(\dfrac{1+x}{1-x} \right)^3$

    B. $y = e^{-(3-2x)^2}$

    C. $y = \log \left(\dfrac{x}{x+2} \right)$

    D. $xy = y^2 + \sin y$

3. (20%) Evaluate the following integrals.

    A. $\displaystyle\int \dfrac{5x-3}{x^2 - 4x + 3} dx$

    B. $\displaystyle\int x \ln(2x) dx$

    C. $\displaystyle\int_0^4 \dfrac{2}{1+3x} dx$

    D. $\displaystyle\int_0^{\pi/2} \sqrt{1 + \cos 4x} \, dx$

4. (10%) Based on the I anitial value $y(0) = 1$, find y that satisfies $\dfrac{dy}{dx} + 2xy + y = 0$.

5. (10%) Find the area under the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.

6. (10%) Show that $\displaystyle\int_0^\infty \int_0^\infty e^{-(x^2 + y^2)} \, dx \, dy$.

7. (10%) Find the radius of convergence and its interval of convergence for the series $\displaystyle\sum_{n=1}^\infty \dfrac{(-1)^{n-1}(3x-1)^n}{n^2}$

統計學、微積分皆出題於同張試題內，請務必擇一科目作答，若兩科均作答，則以統計學的成績計算。

※統計學(100%)

Part I (20%) 是非題, 需說明答題理由.

1. Suppose a researcher is interested in studying the relationship between gross domestic product (GDP) and Internet use (USE) defined as the percent of Internet use in a nation. When the correlation of coefficient equals 0, it means GDP and USE are independent.

2. Consider two events A and B. When $P[A \cap B] = P[A]P[B]$, then two events A and B are mutually exclusive.

3. In the simple linear regression, if the regression slope is positive, the correlation of coefficient has to be positive.

4. An auditing firm was hired to determine if a particular defense plant was overstating the value of their inventory items. When the expected difference (recorded-audited) exceeds 25, the defense plant would be subject to a loss of contract and financial penalties. The hypotheses should be $H_0: \mu \le 25$ versus $H_a: \mu > 25$.

Part II (50%) 單選題

1. In a general social survey, for the question "How many children have you ever had?", the results were

No. Children	0	1	2	3	4	5	6	7	8+
Count	1216	710	1147	738	386	139	83	43	35

Which is the most appropriate graph to display the data? (A) Box-plot (B) Dot plot (C) Histogram (D) Stem-and-leaf plot (E) Scatter plot.

2. Based on an unemployment survey, some of summary measures are obtained for the expected salary in thousand dollars: mean=28, median=30, mode=35. What is the most appropriate shape of the distribution for the expected salary? (A) Bimodal (B) Irregular (C) Left-skewed (D) Right-skewed (E) Symmetric.

3. According to the National Association of Home Builders, the medium selling price of new homes in the United States in January 2007 was $239,800. Which of the following is the most plausible value for the standard deviation? (A) $500 (B) $1,000 (C) 15,000 (D) 60,000 (E) 1,000,000.

4. Consider a sample of 64 female college athletes. Their weight in pounds is measured. Which of the following measure is not appropriate to measure the variability of weight for this data? (A) Empirical rule for a bell-shaped distribution (B) Interquartile (C) Minimum (D) Range (E) Standard deviation

5. Brokerage firms receive awards for their analysts' stockpicking abilities. The following security firms were the top eight firms in terms of investment research for the year

Security firms	Total awards
Salomon Smith Barney	39
Merrill Lynch	28
Morgan Stanley	25
Lehman Brothers	24
Goldman Sachs	22
A. G. Edwards	21
Credit Suisse First Boston	21
J. P. Morgan Chase	20

What is the probability that an award selected at random from the above list is either to Salomon Smith Barney or to Merrill Lynch? (A) 0.055 (B) 0.14 (C) 0.195 (D) 0.25 (E) 0.335.

6. An electronics firm decides to market three different software packages for its personal computers. The marketing analyst gives each of the three packages an 80% chance of success. The outcomes for each of the software packages are independent. What is the probability that only two of packages will be a success? (A) 0.032 (B) 0.096 (C) 0.128 (D) 0.384 (E) 0.512.

7. Continuing 6, suppose the number of successful packages (X) is recorded. What is the probability distribution? (A) Bernoulli distribution (B) Binomial distribution (C) Hypergeometric distribution (D) Geometric distribution (E) Poisson distribution.

8. The funds dispensed daily by an automatic teller machine (ATM) in a grocery store are believed to be normally distributed with a mean of \$3,700 and a standard deviation of \$625. The machine is programmed to notify the store manager if the daily dollar volume is very low (less than or equal to \$200) or unusually high (greater than or equal to \$5,000). What is the percentage of the time will the daily dollar volume not be in either of these two conditions. ? (A) 0.0221 (B) 0.4812 (C) 0.4967(D) 0.9779 (E) 1.

9. The output voltage of power supplies manufactured by Clark Products is believed to follow a normal distribution. Of primary concern to the company is the average output voltage of a particular power supply unit, believed to be 10 volts. A preliminary study found out that the standard deviation for the voltages is 0.7667 volt. How large a sample would they need for sample mean to be within 0.2 volt of the actual average output voltage with 95% confidence? (A) 7 (B) 8 (C) 40 (D) 57 (E) 100.

10. A recent study compared different psychological therapies for teenage girls suffering from anorexia, an eating disorder that can cause them to become dangerously underweight. Each girl's weight was measured before and after a period of therapy. Suppose the investigator would like to know whether the weight changes between two psychological therapies are different statistically significantly. What is the least appropriate test statistic to use? (A) Chi-square test (B) One-way analysis of variance (C) Simple linear regression (D) Two independent sample T test (E) A Wilcoxon test.

Part III (30%) 簡答題

1. (15%) One year a survey asked respondents whether they are a member of an environmental group (variable GRNGROUP) and whether they would be very willing to pay higher taxes to protect the environment (variable GRNTAXES). The following table lists the result:

		Pay higher prices	
		Yes	No
Member of environmental group	Yes	87	32
	No	492	538

   (1) (5%) Compute the estimated probability of being very willing to pay higher taxes to protect the environment given that the person is a member of an environmental group.

   (2) (10%) Choose an appropriate test to examine whether there is any association between GRNGROUP and GRNTAXES. Make statistical inference about the test result.

2. (15%) A consulting firm wished to test if the leadership ability of supervisors, middle-level managers, and upper-level managers differs in a particular company. Using a quantitative measure to record the leadership ability, the consulting firm sampled five individuals and recorded the following data:

Class	Leadership ability
Supervisor	20 23 18 47 22
Middle-level manager	38 42 33 62 23
Upper-level manager	50 57 44 70 35

The following is given some statistical analysis outputs.

Table 1: ANOVA table

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	1590.933333	795.466667	4.55	0.0338
Error	12	2098.000000	174.833333		
Corrected Total	14	3688.933333			

Table 2: Additional analysis for Table 1

R-Square	Coeff Var	Root MSE	Leader Mean
0.431272	33.96179	13.22246	38.93333

   (1) (5%) Write down the hypothesis for this research question and state the statistical analysis that you might consider to use.

   (2) (5%) Regardless of your answer in (1), based on the result in Table 1, what is your finding? Assume that the level of significance=0.05.

   (3) (5%) What is the meaning of R-square in Table 2? Based on the value in Table 2, what is your finding?

Table 1: Area under the standard normal curve

z	$P[Z \leq z]$									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.500	0.504	0.508	0.512	0.516	0.519	0.523	0.527	0.531	0.535
0.10	0.539	0.543	0.547	0.551	0.555	0.559	0.563	0.567	0.571	0.575
0.20	0.579	0.583	0.587	0.591	0.594	0.598	0.602	0.606	0.610	0.614
0.30	0.617	0.621	0.625	0.629	0.633	0.636	0.640	0.644	0.648	0.651
0.40	0.655	0.659	0.662	0.666	0.670	0.673	0.677	0.680	0.684	0.687
0.50	0.691	0.695	0.698	0.701	0.705	0.708	0.712	0.715	0.719	0.722
0.60	0.725	0.729	0.732	0.735	0.738	0.742	0.745	0.748	0.751	0.754
0.70	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.80	0.788	0.791	0.793	0.796	0.799	0.802	0.805	0.807	0.810	0.813
0.90	0.815	0.818	0.821	0.823	0.826	0.828	0.831	0.834	0.836	0.838
1.00	0.841	0.843	0.846	0.848	0.850	0.853	0.855	0.857	0.859	0.862
1.10	0.864	0.866	0.868	0.870	0.872	0.874	0.877	0.879	0.881	0.883
1.20	0.884	0.886	0.888	0.890	0.892	0.894	0.896	0.898	0.899	0.901
1.30	0.903	0.904	0.906	0.908	0.909	0.911	0.913	0.914	0.916	0.917
1.40	0.919	0.920	0.922	0.923	0.925	0.926	0.927	0.929	0.930	0.931
1.50	0.933	0.934	0.935	0.937	0.938	0.939	0.940	0.941	0.942	0.944
1.60	0.945	0.946	0.947	0.948	0.949	0.950	0.951	0.952	0.953	0.954
1.70	0.955	0.956	0.957	0.958	0.959	0.959	0.960	0.961	0.962	0.963
1.80	0.964	0.964	0.965	0.966	0.967	0.967	0.968	0.969	0.969	0.970
1.90	0.971	0.971	0.972	0.973	0.973	0.974	0.975	0.975	0.976	0.976
2.00	0.977	0.977	0.978	0.978	0.979	0.979	0.980	0.980	0.981	0.981
2.10	0.982	0.982	0.983	0.983	0.983	0.984	0.984	0.985	0.985	0.985
2.20	0.986	0.986	0.986	0.987	0.987	0.987	0.988	0.988	0.988	0.989
2.30	0.989	0.989	0.989	0.990	0.990	0.990	0.990	0.991	0.991	0.991
2.40	0.991	0.992	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993
2.50	0.993	0.994	0.994	0.994	0.994	0.994	0.994	0.994	0.995	0.995
2.60	0.995	0.995	0.995	0.995	0.995	0.996	0.996	0.996	0.996	0.996
2.70	0.996	0.996	0.996	0.996	0.996	0.997	0.997	0.997	0.997	0.997
2.80	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.998	0.998
2.90	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998
3.00	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.999	0.999

Table 2: Value of χ_r^2 for chi-square distribution where $P[\chi_r^2 \leq x] = \gamma$

r	0.01	0.025	0.05	0.10	0.5	0.9	0.95	0.975	0.99
1	0.000	0.001	0.004	0.016	0.455	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	1.386	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	2.366	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	3.357	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	4.351	9.236	11.070	12.833	15.086
6	0.872	1.237	1.635	2.204	5.348	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	6.346	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	7.344	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	8.343	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	9.342	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	10.341	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	11.340	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	12.340	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	13.339	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	14.339	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	15.338	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	16.338	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	17.338	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	18.338	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	19.337	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	20.337	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	21.337	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	22.337	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	23.337	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	24.337	34.382	37.652	40.646	44.314