科目：資料結構與演算法 適用：資工系

編號：413

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

The following problems can be answered either in Chinese or in English.

Problem 1 (15%) Let $T(n)$ be a polynomial and $T(n) = T(\frac{3n}{4}) + T(\frac{n}{5}) + n + 1$. Solve $T(n)$.

Problem 2 (15%) The longest increasing subsequence problem is defined as follows: Given an integer sequence $a_1, \ldots, a_n$, find the largest k such that there exist indices $i_1, i_2, \ldots, i_k$ satisfying $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ and $a_{i_1} < a_{i_2} < \cdots < a_{i_k}$. Show how to solve the longest increasing subsequence problem in time $O(n^2)$.

Problem 3 A binary heap is a way to realize a priority queue. Suppose we want to use an array to implement a binary heap. Given i as the index of a node, write down simple expressions in terms of i for the indices of

1. (5%) the parent node of i;
2. (5%) the left child of i;
3. (5%) the right child of i

if they do exist.

Problem 4 (15%) Given ten clauses

$$x_1 \vee x_2 \vee x_4, \quad x_3 \vee \neg x_5 \vee \neg x_6, \quad \neg x_1 \vee \neg x_3 \vee \neg x_5, \quad \neg x_2 \vee x_4 \vee x_5,$$
$$\neg x_1 \vee x_2, \quad \neg x_2 \vee x_3, \quad \neg x_3 \vee x_1, \quad \neg x_4 \vee x_5, \quad \neg x_5 \vee x_6, \quad \neg x_6 \vee x_4,$$

show that their conjunction (that is, their logical-and) is unsatisfiable.

Problem 5 (20%) In the sorting problem, we are given n numbers and have to list these numbers from the smallest to the largest in non-decreasing order. Show how to apply divide-and-conquer to solve the sorting problem in time $O(n \log n)$.

Problem 6 (20%) Suppose we know that sorting n numbers by comparisons requires $\Omega(n \log n)$ time. Show how to use this fact to prove that constructing the convex hull for n points in the plane has time complexity $\Omega(n \log n)$.