1. 假設迴歸模型如下：$y_i = \beta x_i + e_i$, $i=1,2,...,n$, 且 $e_i \sim \text{IID } N(0,\sigma^2)$, x_i 為非隨機。回答以下問題：

   (a) 導出 β 的最小平方估計式 $\hat{\beta}$.

   (b) 導出 β 與 σ^2 的最大概似估計式，分別以 $\hat{\beta}_m$ 與 $\hat{\sigma}_m^2$ 表示。

   (c) 證明 $\hat{\beta}_m$ 是 β 的不偏估計式；但 $\hat{\sigma}_m^2$ 是 σ^2 的偏誤估計式。(每小題 10 分)

2. 假設家庭所得滿足以下迴歸模型：$FI_i = \alpha + \beta_1 HE_i + \beta_2 WE_i + \varepsilon_i$, $i=1,2,...,n$, 其中 FI, HE, WE 分別表示家庭所得，丈夫受教育年數，妻子受教育年數。若某生設定的計量模型為：$FI_i = \alpha + \beta_1 HE_i + e_i$, 並以最小平方法進行估計，分別以 $\hat{\alpha}$ 與 $\hat{\beta}_1$ 表示 α 與 β_1 的估計值。請問 $\hat{\beta}_1$ 高估或者低估 β_1？必須說明理由。(10 分)

3. 假設迴歸模型如下：$y_i = \beta x_i + e_i$, $i=1,2,..,n$, $V(e_i) = \sigma_i^2 = \sigma^2 x_i$, $x_i > 0$. 請導出加權最小平方估計式 (weighted least squares estimator). (10 分)

4. **(10%)** Players A and B toss a fair dice in order. The first one to throw "six" wins. What are their respective chances of winning?

5. **(10%)** Suppose that the distribution of scores on a statistics midterm exam of NCNU students has mean 75 and standard deviation 10. What is the upper bound for the probability that a student failed in this exam?

6. **(30%)** A mobile pone company claims that at most 10% of its product needed any type of repair in the first two years. I a sample of 10000 customers, 1100 needs repair in the first two years. To check whether this data disagrees with the claim of this company, please complete the following hypothesis-testing procedure. (Hint: Using Central Limit Theorem, a binomial distribution can be approximated by normal distribution when the sample size is large enough.)

(1) State the null hypothesis and alternate hypothesis.

(2) Select the proper test statistic.

(3) Let the level of significance be 0.05, formulate the decision rule.

(4) Make decision and explain your answer.

Let z is standard normal random variable, we have

$P(z \le 0.54) = 0.7$, $P(z \le 0.67) = 0.75$, $P(z \le 0.85) = 0.8$, $P(z \le 1.03) = 0.85$,

$P(z \le 1.28) = 0.9$, $P(z \le 1.64) = 0.95$, $P(z \le 1.96) = 0.975$, $P(z \le 2.57) = 0.99$.