

## 國立臺灣科技大學 104 學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組  
 科目：工程數學

(總分為 100 分)

1. Given  $y'' + 6y' + 9y = 10\sin x$ , (20%)  
 (a) Find its homogeneous solution; (10%)  
 (b) Find its particular solution. (10%)

2. Solve the initial value problem  $y' + 4y = 1$ ;  $y(0) = -1$ , using **only Laplace transform**. (20%)

3. Let  $A$  be the  $3 \times 3$  matrix defined as follows: (20%)

$$A = \begin{bmatrix} 0 & -1/4 & -1/2 \\ 1 & 1 & 1 \\ 1/2 & 1/4 & 1 \end{bmatrix}$$

- (a) Find the matrix  $S$  that *diagonalizes* the matrix  $A$ , i.e., find the matrices  $S$  and  $A$  such that  $A = SAS^{-1}$ . (5%)  
 (b) Calculate  $\lim_{n \rightarrow \infty} A^n = ?$  (5%)

- (c) Let  $\vec{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}$  and solve the set of ordinary differential equations

$$\frac{d}{dt} \vec{y} = A\vec{y} \text{ subjected to the following initial conditions: } \vec{y}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

(10%)

4. Let  $\vec{v}(x, y, z) = (xy, y \ln y, 0)$  and  $C$  be the *closed path* enclosing the (20%)  
 region bounded by  $y = x + 1$ ,  $y = 2x - 1$ , and  $y = 1$  and traversing in  
*counterclockwise* direction.

- (a) Determine if the vector field  $\vec{v}(x, y, z)$  is *irrotational*? (5%)  
 (b) Sketch the closed path  $C$ . (5%)

- (c) Calculate the *line integral*  $\oint_C \vec{v} \cdot d\vec{l}$ , where  $d\vec{l}$  is the *line element* along the  
 path  $C$ . (10%)

5. Solve the following boundary value problem (BVP) governed by the (20%)  
*Helmholtz equation*:

$$\nabla^2 \phi + \phi = 0, \quad 0 < x < \pi, \quad 0 < y < \pi,$$

subjected to the following boundary conditions:

$$\phi(x, 0) = \phi(x, \pi) = 0, \quad 0 \leq x \leq \pi$$

and

$$\phi(0, y) = 0, \quad \phi(\pi, y) = \sin y + \sin 2y, \quad 0 \leq y \leq \pi.$$

