

科目：微積分 適用：國企系乙組

編號：331

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. State and prove the Fundamental Theorem of Calculus. (20%)

2. Prove or give a counter example for the following statements:

(2.1) Suppose both a_n and b_n are a series of real numbers.

If $\lim_{n \rightarrow \infty} (a_n + b_n) = 0$, then $\lim_{n \rightarrow \infty} a_n = 0$ and $\lim_{n \rightarrow \infty} b_n = 0$. (5%)

(2.2) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \rightarrow \infty} a_n = 0$. (5%)

(2.3) If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges. (5%)

(2.4) If the function $f(x)$ is continuous at every point of E , then $f(x)$ is differentiable at every point of E . (5%)

(2.5) If the function $f(x)$ is differentiable at every point of E , then $f(x)$ is continuous at every point of E . (10%)

3. Find $\frac{dy}{dx}$ for the following functions.

(3.1) $y^3 + y^2 - 5y - x^2 = 4$ (5%)

(3.2) $\frac{x+y}{2x-y} = 1$ (5%)

4. Find the derivative of the functions.

(4.1) $f(x) = \ln[x(x^2 + 1)^2]$ (5%)

(4.2) $f(x) = \frac{x^2}{\ln(x)}$ (5%)

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5. Find the integral.

$$(5.1) \int x^3 \ln(x) dx \quad (5\%)$$

$$(5.2) \int_0^x \frac{x}{\sqrt{x-1}} dx \quad (5\%)$$

6. Evaluate the integral.

$$(6.1) \int_1^e 2x \ln(x) dx \quad (5\%)$$

$$(6.2) \int_0^5 \frac{x}{(4+x)^2} dx \quad (5\%)$$

7. The revenue (in dollars per year) for a new product is modeled by

$$R = 10,000 * \left[1 - \frac{1}{(1 + 0.17t)^{0.5}} \right],$$

where t is the time in years. Estimate the total revenue from sales of the product over its first 2 years on the market. (10%)