

國立臺北科技大學 104 學年度碩士班招生考試

系所組別：1201、1202、1203 製造科技研究所

第二節 微分方程 試題

第一頁 共一頁

**注意事項：**

1. 本試題共五題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve the following differential equation (20%):

(1)  $x^2y'' - 5xy' + 10y = 0, y(1) = 2, y'(1) = 9, (10\%)$

(2)  $(\sin(y) \cos(y) + x \cos^2(y))dx + xdy = 0, (10\%)$

2. Using Laplace transform to solve the following differential equation (20%):

$$y'' + 4y = f(x), y(0) = y'(0) = 0$$

$$f(x) = \begin{cases} 0, & x < 2 \\ x, & x \geq 2 \end{cases}$$

3. Solve the following problem (20%):

$$y'' - y = e^x$$

(1) using the undetermined coefficient method (10%)

(2) using the variation parameter method(10%)

4. Consider a temperature distribution  $u(x, t)$  in a thin homogeneous bar of length  $L$  and conductivity  $k$  given the initial temperature in the bar at time zero in the cross section at  $x$  perpendicular to the  $x$ -axis is  $f(x)$ . The ends of the bar are maintained at temperature zero for all time. (20%):

(1) Please model this problem as a boundary value problem (10%)

(2) Please solve the solution of  $u(x, t)$  using separation of variables (10%)

5. Find the solution of the differential equation

$$2xy''(x) + (2x + 1)y'(x) + 2y(x) = 0$$

near  $x = 0$  (20%).