注意事項：

1. 本試題共五題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. Solve the following differential equation (20%):

(1) $x^3 y' + x^2 y = 2y^{-4/3}$ (10%)

(2) $y'' + y = x\sin(x)$ using undetermined coefficient method (10%)

2. Using Laplace transformation to solve the initial value problem (20%):

$$y'' + 8y' + 12y = f(x), \qquad y(0) = y'(0) = 0$$

$$f(x) = \begin{cases} 0, & x < 3 \\ x, & x \geq 3 \end{cases}$$

3. Determined the general solution using Gauss elimination (20%):

$$\begin{cases} x_1 - x_2 + x_3 - x_4 = -2 \\ -2x_1 + 3x_2 - x_3 + 2x_4 = 5 \\ 4x_1 - 2x_2 + 2x_3 - 3x_4 = 6 \end{cases}$$

4. Consider heat conduction in a long bar with length L and conductivity k. Both of the ends are insulated. If the initial temperature is $f(x)$ (20%) :

(1)Model this problem as a boundary value problem (10%)

(2)Solve $u(x,t)$ using separation variable method (10%)

5. Evaluate the following integral by the contour integral and the Cauchy integral theorem (20%)

$$\int_0^\infty \frac{\cos(x)}{1 + x^4} dx$$