

科目：統計學 適用：財金系

編號：353

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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第 1 頁

【請依題號依序作答並列出過程，無過程者不予計分！】

1. (5%) Let X_1, X_2, \dots, X_n be independent random variables which all have the same probability distribution, with mean μ and variance σ^2 . Let

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

- (a) Use the properties of expected values to show that $E(\bar{X}) = \mu$. (2%)
 (b) Use the properties of variance to show that $\text{var}(\bar{X}) = \sigma^2/n$. How have you used the assumption of independence? (3%)

2. (10%) Let X be a uniform random variable on the interval (a, b) .

- (a) Use integration techniques to find the mean and variance of X . (6%)
 (b) Find the cumulative distribution function of X . (4%)

3. (10%) Let X have a normal distribution with mean μ and variance σ^2 . Use the change of variable technique to find the probability density function of $Y = aX + b$.

4. (15%) Suppose that the joint pdf of the continuous random variables X and Y is $f(x, y) = 6x^2y$ for $0 \leq x \leq 1, 0 \leq y \leq 1$.

- (a) Does this function satisfy the conditions for a valid pdf? (3%)
 (b) Find the marginal pdf of X , as well as its mean and variance. (4%)
 (c) Find the conditional pdf of X given $Y = 1/2$. (3%)
 (d) Are X and Y independent? Explain. (5%)

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5. (10%) Suppose that the economic profitability of a new supermarket depends on households spending more than \$5.5 out of each additional \$100 weekly income on food and that construction will not proceed unless there is strong evidence to this effect. If $\beta \leq 5.5$, then the supermarket will be unprofitable and the owners would not want to build it. The least squares estimate of β from $N = 1000$ observations is $b = 10.21$ with standard error $se(b) = 2.09$. Based on the available data, answer the following questions.
- (a) What will the null and alternative hypothesis be? (2%)
 - (b) Specify the test statistic and its distribution if the null hypothesis is true. (2%)
 - (c) For the $\alpha = 0.05$ level of significance, determine the rejection region. (2%)
 - (d) Calculate the sample value of the test statistic. (2%)
 - (e) State your conclusion. (2%)
6. (10%) To evaluate alternative retirement benefit packages for its employees, a large corporation must determine the mean age of its workforce. Assume that the age of its employees is normally distributed. Since the corporation has thousands of workers, a simple is to be taken. If the standard deviation of ages is known to be $\sigma = 21$ years, how large should the sample be to ensure that a 95% interval estimate of mean age is no more than four years wide?

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7. (25%) A classical linear regression model (CLRM) can be written as

$$y = X\beta + u$$

where y is of dimension $T \times 1$; X is of dimension $T \times k$; β is of dimension $k \times 1$; u is of dimension $T \times 1$.

- (a) What assumptions are usually made about the unobservable error term u in the CLRM? Briefly explain the meaning of each. (5%)
- (b) Derivation of the ordinary least squares (OLS) estimator for the parameter β . (8%)
- (c) OLS and maximum likelihood are used to estimate the parameters of a standard linear regression model. Will they give the same estimates? Explain your answer. (7%)
- (d) What would the consequence be for a regression model if the error term were not 'homoscedastic'? (5%)

8. (15%) A researcher estimates the following two regression models

$$(1) \quad y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t$$

$$(2) \quad y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + v_t$$

where u_t and v_t are iid disturbances.

- (a) Assume that x_{3t} is an irrelevant variable which does not enter into the data generating process for y_t . Will the value of (i) R^2 , (ii) Adjusted R^2 , be higher for the second model than the first? Explain your answers. (5%)
- (b) Why is it desirable to remove insignificant variables from a regression? (5%)
- (c) When using the OLS estimation method, the problem of 'multicollinearity' occurs if the variables x_{2t} and x_{3t} in Eq. (2) are highly correlated. What will happen if the 'multicollinearity' is present but ignored? (5%)