

國立彰化師範大學104學年度碩士班招生考試試題

系所： 電子工程學系(甲組選考甲、乙組選考甲)
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資訊工程學系(選考戊)

科目： 工程數學

☆☆請在答案紙上作答☆☆

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1. Let $f(x)$ and $f'(x)$ be piecewise continuous on the interval $\left(-\frac{T}{2}, \frac{T}{2}\right)$. This function can be expanded as $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\omega x}$, where $\omega = \frac{2\pi}{T}$.
- (a) Let $f(x) = |\cos x|$, find the coefficient c_n . (10%)
- (b) Show that the Parseval's identity $\frac{1}{T} \int_{-T/2}^{T/2} |f(x)|^2 dx = \sum_{n=-\infty}^{\infty} |c_n|^2$. (10%)

2. Use Cauchy's residues theorem to evaluate the given integral along the indicated contour. (15%)

$$\oint_C \left[ze^{3/z} + \frac{\sin z}{z^2(z-\pi)^3} \right] dz, \quad C: |z|=6$$

3. Find the eigenvalues and the corresponding eigenfunctions of the Sturm-Liouville problem. (15%)

$$y'' + \lambda y = 0, \quad y(0) = y'(L) = 0$$

4. Consider the given linear system $\dot{x}_1 = x_2$
 $\dot{x}_2 = -4(x_2 + x_1) + u(t)$,

where $(\cdot) = \frac{d}{dt}$, $x_1 = x_1(t)$, $x_2 = x_2(t)$, and $u(t)$ is a unit step. The initial conditions are: $x_1(0) = 0$ and $\dot{x}_1(0) = 0$. It is known that the given system can be expressed in a state vector equation, which is:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}(t), \quad \mathbf{x} = (x_1, x_2)^T.$$

Please answer the following questions:

- (a) Identify the matrices \mathbf{A} , and $\mathbf{b}(t)$. (10%)
- (b) Find $\mathbf{x}(t)$ by taking the Laplace transform to the state vector equation. (20%)
- (c) Express the given system as a second-order differential equation. Please find the solution of x_1 and x_2 by solving the differential equation in time domain directly. $u(t)$ is a unit step. (20%)

Note:

1. Please be careful about the form that the problem was asked.
2. Please show all the details as many as possible; your grade is based on your procedures. The Laplace

transform of a unit step is $\mathcal{L}[u(t)] = \frac{1}{s}$.