國立彰化師範大學104學年度碩士班招生考試試題

系所： 電信工程學研究所(選考戊)、資訊工程學系(選考辛)

科目： 線性代數與機率

☆☆請在答案紙上作答☆☆

1. Find the projection vector from (3, 4) to (-1, 7). **(5%)**

2. Find the exact condition on (*a, b, c*) so that the following linear system is consistent.

$$\begin{cases} 3x_1 + x_2 + x_3 = a \\ -3x_1 + 9x_2 - 5x_3 = b \\ 6x_1 - 3x_2 + 4x_3 = c \end{cases}$$ **(5%)**

3. Find $\boldsymbol{A}^{-1}$ if matrix $\boldsymbol{A} = \frac{1}{7}\begin{bmatrix} 2 & 3 & 6 \\ 3 & -6 & 2 \\ 6 & 2 & -3 \end{bmatrix}$. **(5%)**

4. Compute $det(\boldsymbol{B}^5)$, where $\boldsymbol{B} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$. **(5%)**

5. Let $\boldsymbol{C} = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}$; (a) find the *eigenvalues* of ***C***, (b) find the *eigenvectors* of ***C.*** **(10%)**

6. Let $\boldsymbol{D} = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 3 & 10 & -7 & 1 \\ -5 & -5 & 3 & 7 \end{bmatrix}$; find the ***null space*** of ***D***, $N(\boldsymbol{D})$. **(10%)**

7. Find the *transition matrix* corresponding to the change of basis from [$\mathbf{v}_1$, $\mathbf{v}_2$] to [$\mathbf{u}_1$, $\mathbf{u}_2$], where $\mathbf{v}_1=[1, -2]^T$, $\mathbf{v}_2=[1, -1]^T$ and $\mathbf{u}_1=[3, 1]^T$, $\mathbf{u}_2=[5, 2]^T$. **(10%)**

8. In throwing a fair dice, the player wins the amount that the dice shows if the result is even and loses that amount if it is odd.
   (a) What is the probability space of this random experiment? **(8%)**
   (b) Please model this random experiment using a random variable. **(8%)**

9. Let X be a random variable with mean μ and variance σ^2. Please state and prove the Chebyshev's inequality. **(16%)**

10. Let X be a random variable that is uniformly distributed over the region [0,1]. What is the mean and the variance of X? **(18%)**