國立彰化師範大學104學年度碩士班招生考試試題

系所：物理學系（甲組選考甲）、光電科技研究所（選考甲）　　科目：工程數學

☆☆請在答案紙上作答☆☆　　共 1 頁，第 1 頁

1. y_p is a solution of the given equation. Solve the given initial value problem.

$$y''+4y=-12\sin 2x,\quad y(0)=1,\quad y'(0)=3;\quad y_p=3x\cdot\cos 2x \qquad (15\%)$$

2. (a) Find the Fourier transform of the function $f(x)$.

$$f(x)=\begin{cases}-1 & if \quad -1<x<0\\ 1 & if \quad 0<x<1\\ 0 & otherwise\end{cases} \qquad (15\%)$$

(b) Find the Fourier series of the function $f(x)$, and period= $p=2L=2$.

$$f(x)=3x^2 \quad (-1<x<1) \qquad (10\%)$$

3. Find the Laplace transforms of $e^t\sin t$. (10%)

4. Define the matrices A and Λ as

$$\mathrm{A}=\begin{bmatrix}-3 & 0 & 0\\ 1 & -2 & 0\\ 0 & 0 & -4\end{bmatrix} \text{ and } \Lambda=\begin{bmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{bmatrix}, \text{ respectively.}$$

(a) If Λ is the diagonal matrix of A, find the values of *a*, *b*, and *c* for $a\le b\le c$. (6%)

(b) Find a matrix X such that $\Lambda=\mathrm{X}^{-1}\mathrm{AX}$. (9%)

(c) Find a matrix B such that $\mathrm{B}^3=\mathrm{A}$. (5%)

5. (a) Find the value of $\int_c \frac{3z^3+2}{(z-1)(z^2+9)}dz$, where *c* is the path taken counterclockwise around the circle $|z-2|=2$. (10%)

(b) Evaluate the integral. $I=\int_{-\infty}^{\infty}\frac{x^3\sin(ax)}{x^4+4}dx$. (10%)

6. Let $\vec{F}(x,y,z)=z\hat{j}+z\hat{k}$ represent the flow of a liquid. Find the flux of $\vec{F}$ through the surface *S* given by that portion of the plane $3x+2y+z=6$ in the first octant oriented upward. (10%)