國立彰化師範大學104學年度碩士班招生考試試題

系所： 數學系　　　　科目： 線性代數

☆☆請在答案紙上作答☆☆　　　　共 **1** 頁，第 **1** 頁

1. Let $A=\begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & s-1 \\ 3 & s+1 & 3 \end{bmatrix}$. Find all real number s so that $\text{rank}(A)=2$. (10%)

2. Prove that $1, \dfrac{1}{x-1}, \dfrac{1}{(x-1)^2}$ are linearly independent. (10%)

3. Suppose $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3\}$ is a basis for a vector space V. Let

$$\mathbf{w}_1=\mathbf{v}_1-\mathbf{v}_2, \mathbf{w}_2=\mathbf{v}_2-\mathbf{v}_3, \mathbf{w}_3=\mathbf{v}_1+\mathbf{v}_2+\mathbf{v}_3$$

   Prove that $\{\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3\}$ is a basis for V. (15%)

4. Let A be an 3×3 matrix having eigenvalues $1, -1$. Suppose $[1,2,-1]$ is an eigenvector corresponding to the eigenvalue 1 and $[1,0,1], [-1,1,1]$ are eigenvectors corresponding to the eigenvalue -1. Find a matrix P so that $P^{-1}AP$ is a diagonal matrix and then determine A. (15%)

5. Let $P_2=\{a_0+a_1x+a_2x^2 \mid a_0,a_1,a_2\in R\}$, and let $T:P_2\to P_2$ be defined by $T(p(x))=p(1)+p'(0)x+[p'(0)+p''(0)]x^2$. (20%)
   (i) Show that T is a linear transformation.
   (ii) Determine whether T is invertible, and compute T^{-1} if it exists.
   (iii) Determine whether T is diagonalizable.

6. Prove that similar matrices have the same trace and the same determinant. (10%)

7. Let be a linear operator on an inner product space V, and let W be a T-invariant subspace of V. Prove that : (20%)
   (i) $W^{\perp}$ is T^*- invariant.
   (ii) If W is both T- and T^*- invariant, then $(T_W)^*=(T^*)_W$.