科目：**工程數學**　　　　　　　　　　適用系所：**機電工程學系–光機電系統組**

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. Find and graph the output response V₀(t) to the input Vᵢ(t) shown in Fig. 2 for the following RC circuit shown in Fig. 1, where RC=1 ms(0.001 sec.) (18 分)
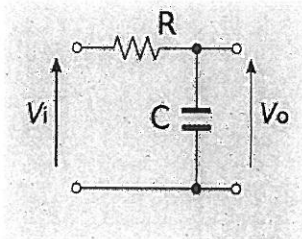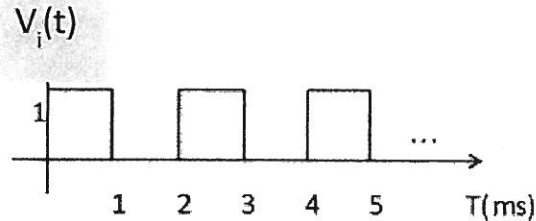

Fig. 1

Fig. 2

2. Find and graph the output response y(t) to the unit step input μ(t) for the following differential equation. (18 分)

$$y^{''}(t) + 8y^{'}(t) + 100y(t) = 100u(t), \quad y(0) = 0, \ y^{'}(0) = 0$$

3. Obtain the general solution for the following equation

$$y'' - y' = 5\sin(2t) \quad (14 分)$$

4. Is there any 2×2 matrix, $\mathbf{A}$, with real entries such that $\mathbf{A}^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$? Give a detailed explanation. (10 分)

5. Set up the system of equations for the mesh currents, i_1 and i_2, in the network shown in Figure 1. (15 分)

(a) Express the system as a matrix equation, $\mathbf{AX} = \mathbf{B}$, in which $\mathbf{X} = \begin{bmatrix} i_1 & i_2 \end{bmatrix}^{\mathrm{T}}$.

(b) Show that the coefficient matrix **A** is nonsingular.

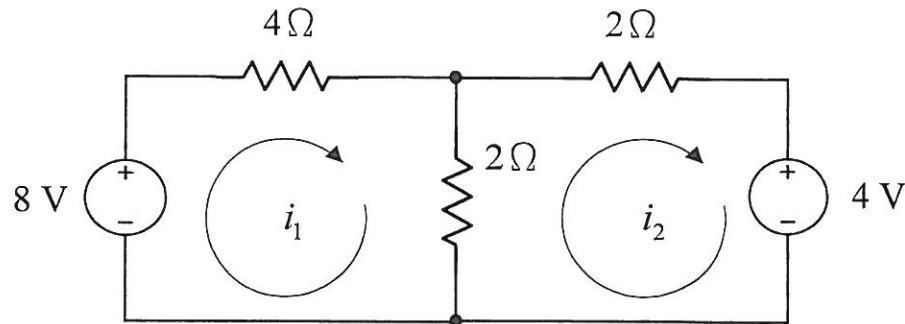**(c)** Use $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ to solve for the currents.

Figure 1. Network for Problem 2

6. The electric field **E** generated by a point charge q at the origin can be obtained from Coulomb's law, or Coulomb's inverse-square law:

$$\mathbf{E} = kq\mathbf{r}/\|\mathbf{r}\|^3,$$

in which k is a constant and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Determine the electric flux out

of a sphere $x^2 + y^2 + z^2 = 4$ due to a point charge q. (15 分)

Note: The surface area of a sphere of radius a is $4\pi a^2$.

7. Consider two functions, $f_1(x) = 1$ and $f_2(x) = x$, which are orthogonal with

respect to the weight function $w(x) = 1$ on the interval $\begin{bmatrix} -2 & 2 \end{bmatrix}$. Find

constants c_1 and c_2 such that the function $f_3(x) = x + c_1 x^2 + c_2 x^3$ is

orthogonal to both $f_1(x)$ and $f_2(x)$ on the same interval. (10 分)