國立臺灣師範大學104學年度碩士班招生考試試題

科目：工程數學　　適用系所：機電工程學系- 精密機械組

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

【試題 1】(15 分)

To solve $\cos y\,dx - \cos y\,dy - 2(x-y)\sin y\,dy = 0$

【試題 2】(20 分)

Solve the initial value problem using the method of Laplace transformations:

$$\begin{cases} z'' + y' = \cos x \\ y'' - z = \sin x \end{cases}$$

and $z(0) = -1,\ z'(0) = -1,\ y(0) = 1,\ y'(0) = 0$

【試題 3】(15 分)

Evaluate $I = \iint_S (x^3 dydz + x^2 ydzdx + x^2 zdxdy)$, where S is the closed surface consisting of the cylinder $x^2 + y^2 = 4$ and the circular disks z =0 and z =1.

【試題 4】(15 分)

To solve by matrix method

$$y_1' = 10y_1 - 10y_2 - 4y_3$$
$$y_2' = -10y_1 + y_2 - 14y_3$$
$$y_3' = -4y_1 - 14y_2 - 2y_3$$

【試題 5】(20 分)

Consider the following boundary value problem:

$$x^2 y'' + 3xy' + \lambda y = 0,\ y(1) = y(e^2) = 0$$

(1) Show that the above equation is a Sturm-Liouville differential equation.

(2) Find the eigenvalues and the corresponding eigenfunctions.

【試題 6】(15 分)

Use the Laplace transform to solve

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2},\ 0 < x < t,\ t > 0$$

$$u(0,\ t) = 0,\ \ u(3,\ t) = 0,\ \ t > 0$$

$$u(x,\ 0) = \sin\frac{\pi x}{3},\ \ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$$