國立臺灣師範大學 104 學年度碩士班招生考試試題

科目：**應用數學**　　適用系所：**物理學系**

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

Each problem is 10 points.

1. Assuming $|x|, |y| \ll 1$, use the Taylor expansion to expand $f(x, y)$ with respect to $(x, y) = (0, 0)$ to the second order of x, y.

2. $f(x) = \frac{\cosh x}{\sinh x} - \frac{1}{x}$, $|x| \ll 1$. Expand $f(x)$ in powers of x and find out the leading non-zero term.

3. Evaluate the integral $\int_0^\infty dx\delta\left(x^2 - a^2\right)\sqrt{x}$, in which $\delta()$ is the Dirac delta function, and a is a constant.

4. Evaluate $e^{i\alpha M}$, where α is a real number, and $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

5. $\vec{F}(x, y, z) = y\hat{x} + x\hat{y} + z\hat{z}$, where $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors of a rectangular coordinate. S is the surface of a cube (with side-length 1) centered at the origin. Evaluate the surface integral $\int_S \vec{F} \cdot d\vec{a}$.

6. $H = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. It can be diagonalized by a unitary transformation, $H = U^{-1}DU$, where D is a diagonal matrix, and U is an unitary matrix. First, find out the matrix D.

7. Follow Prob. 6, find out the unitary matrix U.

8. $f(x) = e^{-x^2/2}e^{ik_0x}$, k_0 is a constant. Find out its Fourier transformation $\tilde{f}(k) = \frac{1}{\sqrt{2\pi}}\int dx e^{-ikx} f(x)$.

9. Differential equation $\frac{dy(x)}{dx} = xy$, and $y(0)$=1. Find out $y(1)$.

10. Assume $a > |b|$, evaluate $\int_0^{2\pi} \frac{d\theta}{a + b\sin\theta}$.