

國立臺灣師範大學 104 學年度碩士班招生考試試題

科目：工程數學

適用系所：光電科技研究所

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. Please find the general solution for the simple system of linear first-order equations: (10 分)

$$\frac{dx}{dt} = 3y, \quad \frac{dy}{dt} = 2x.$$

2. Find the general solution of the equation (10 分)

$$y''' + y'' = e^x \cos x$$

3. Find the eigenvalues and eigenvectors of matrix $A = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix}$. (10 分)

4. Find an equation of the tangent plane to the graph of $x^2 - 4y^2 + z^2 = 16$ at $(2, 1, 4)$. (10 分)

5. Find the work done by a force $\mathbf{F} = x\hat{i} + y\hat{j}$ along the curve C traced by $\mathbf{r}(t) = \hat{i} \cos t + \hat{j} \sin t$ from $t = 0$ to $t = \pi$. (10 分)

6. Use the polar coordinates to evaluate $\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5+x^2+y^2} dy dx = ?$ (10 分)

7. Find the inverse Laplace transform: $L^{-1} \left\{ \frac{1}{(s^2 + k^2)^2} \right\} = ?$ (k is a constant). (10 分)

8. Verify that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane. Next, try to find the conjugate harmonic function of u . (10 分)

9. Evaluate the integral $\oint_C \frac{5z+7}{z^2+2z-3} dz$, where C is the circle $|z-2|=2$. (10 分)

10. Show that $\sin^{-1} z = -i \ln \left[iz + (1-z^2)^{1/2} \right]$. Find all values of $\sin^{-1}(\sqrt{5})$. (10 分)