

科目：工程數學 適用：土木系(大地、水利及防災組)

編號：424

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題
共壹頁
第壹頁

1. (20%) Solve the ordinary differential equation $y'' + x^2y = 0$ using the power series method.

2. (20%) Use the Laplace transform approach to solve the ordinary differential equation $y'' + 16y = 1/t$ with the initial conditions $y(0) = 0$ and $y'(0) = 0$.

3. (20%) Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} -3 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

4. (20%) Solve the linear system of equations

$$-x_1 + 2x_2 - 3x_3 + x_4 = 1$$

$$x_1 - x_2 + 2x_3 - 2x_4 = -4$$

$$x_1 + x_2 - 4x_3 + 4x_4 = 6$$

$$-2x_1 + 3x_2 - 2x_3 + x_4 = 1$$

5. (20%) Assume that $f = f(x, y, z)$, which is a scalar function; $\vec{v}(x, y, z)$ is a vector function: $\vec{v}(x, y, z) = v_1(x, y, z)\vec{i} + v_2(x, y, z)\vec{j} + v_3(x, y, z)\vec{k}$. Prove

that (a) $\nabla \times (\nabla f) = \vec{0}$; (b) $\nabla \cdot (\nabla \times \vec{v}) = 0$.