大同大學 104 學年度研究所碩士班入學考試試題

考試科目：基本數學　　所別：資訊工程研究所　　

註：本次考試 不可以參考自己的書籍及筆記； 不可以使用字典； 不可以使用計算器。

Part I: Linear Algebra (50 points)

True (T) or False (F) [20%]:
2 points for each correct answer, and −1 point for each wrong answer. Be careful.

1. If A and B are square matrices of order n such that $\det(AB) = -1$, then both A and B are nonsingular.
2. Let A be a 3×2 matrix and B a 2×3 matrix. Then the determinant of AB must be zero.
3. If A is an invertible $n \times n$ matrix, then A^2 must also be invertible.
4. The plane $x + 4y - 2z = 9$ is a subspace of R^3.
5. If A is a square matrix such that $A^2 - A = 0$, then $A = I$ or $A = 0$.
6. Let A and B be invertible matrices. If A is similar to B, then A^{-1} must be similar to B^{-1}.
7. Eigenvalues must be nonzero scalars.
8. Similar matrices always have the same eigenvalues.
9. Similar matrices always have the same eigenvectors.
10. Two distinct eigenvectors corresponding to the same eigenvalue are always linearly dependent.

Muliple Choice [30%]:
Each of the following questions has axactly one correct choice. 3 points for each correct choice, and −1 point for each wrong choice. Be careful.

1. The determinant of $\begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 5 & 4 \end{bmatrix}$ equals

   (a) −120 (b) −24 (c) 0 (d) 24 (e) 120

2. What are the eigenvalues of $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$?

   (a) $\lambda_1 = 2, \lambda_2 = -5$.
   (b) $\lambda_1 = -2, \lambda_2 = 5$.
   (c) $\lambda_1 = 2, \lambda_2 = 5$.
   (d) $\lambda_1 = -2, \lambda_2 = -5$.
   (e) Not available.

3. The number of linearly independent eigenvectors of $M = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 5 & 5 \end{bmatrix}$

   (a) 1 (b) 2 (c) 3 (d) 4 (e) none.

4. Rank of the matrix $\begin{bmatrix} 21 & -7 & 0 & 0 & 0 \\ -11 & 9 & 0 & 0 & 9 \\ 0 & -19 & 35 & 0 & 0 \\ 0 & 15 & 0 & 12 & 20 \\ 0 & 0 & -24 & 21 & 35 \end{bmatrix}$ is

   (a) 2 (b) 5 (c) 4 (d) 5 (e) none.

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5. The sum of eigenvalues of $\begin{bmatrix} -1 & -2 & -1 \\ -2 & 3 & 2 \\ -1 & 2 & -3 \end{bmatrix}$ is

   (a) −1 (b) −3 (c) 1 (d) 3 (e) none.

6. The system of homogenous linear equations $\begin{bmatrix} 4d-1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 4d-1 \end{bmatrix} = 0$ has a non-trial solution if d equals

   (a) −1/2 (b) 1/2 (c) −1/4 (d) 1/4 (e) none.

7. A set of linear equations is represented by the matrix $Ax = b$. The necessary condition for the existence of a solution for this system is
   (a) A must be invertible
   (b) B must be linearly depended on the columns of A
   (c) B must be linearly independed on the columns of A
   (d) B must be linearly depended on the rows of A
   (e) none of the above.

8. Eigenvalues of a square sysmmetic matrix are always
   (a) positive
   (b) real and imaginary
   (c) negative
   (d) real
   (e) none of the above.

9. Which of the following is not a linear transformation:

   (a) $T(x, y, z) = (x, 2y, 3x - y)$
   (b) $T(x, y, z) = (x - y, 0, y - z)$
   (c) $T(x, y, z) = (0, 0, 0)$
   (d) $T(x, y, z) = (1, x, z)$
   (e) none of the above.

10. If $T: U \rightarrow V$ is any linear transformation from U to V and $B = \{u_1, u_2, \ldots, u_n\}$ is a basis for U, then $T(B) = \{T(u_1), T(u_2), \ldots, T(u_n)\}$

    (a) spans V
    (b) spans U
    (c) is a basis for V
    (d) is linearly independent
    (e) none of the above.

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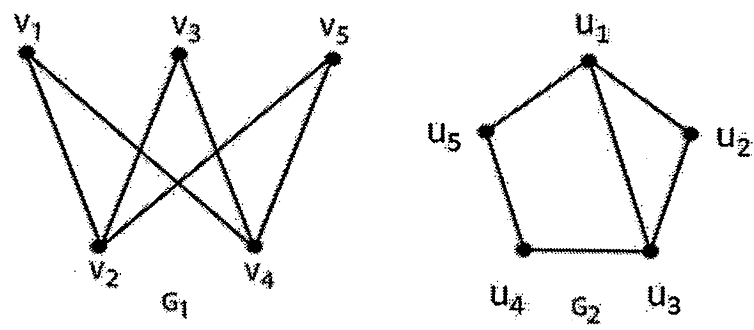
Part II: Discrete Math (50 points)

1. (5%) Are these two graphs isomorphic? If so, determine the mapping between vertices. Otherwise, state your reasons.

V_1	V_2	V_3	V_4	V_5



2. (5%) Let A={a, {a}}, B={a, b, c} Find Cartesian product of (A-B) and B.

3. (5%) Draw the Hasse diagram for the partial ordering {(A, B)| A $\subseteq$ B } on the power set *P(S)* where *S* = {a, b, c}.

4. (5%) Let G be an undirected graph with *X* components, *Y* vertices and *Z* edges. Prove that $Z \geqq Y-X$.

5. (20%) Let F_n be the Fibonacci number. $F_0 = F_1 = 1$

   (a). Solve the recurrence relation for the Fibonacci sequence, where it is defined as:
   $F_n = F_{n-1} + F_{n-2}$

   (b). Prove that $\sum_{k=1}^{n} F_k^2 = F_n F_{n+1}$, for all $n \geqq 0$.

6. (10%) *True or False;* explain your answers. N:Natural numbers,R: Real numbers.
   In the questions (a)-(b) determine whether the rule defines a function with the given domain and codomain.

   (a). $F: \mathbf{R} \to \mathbf{R}$ where $F(x) = \frac{1}{x-5}$.

   (b). Suppose $f: \mathbf{N} \to \mathbf{N}$ where $f(n) = 4n^2 + 1$. Determine whether f is 1-1 (one to one).