

逢甲大學104學年度碩士班考試入學試題

編號：052 科目代碼：323

科目	線性代數與機率	適用 系所	通訊工程學系	時間	100 分鐘
----	---------	----------	--------	----	--------

※請務必在答案卷作答區內作答。

共2頁第1頁

1. Let

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

- (a) (6%) Find the eigen values of A .
- (b) (6%) Find the eigen vectors of A .
- (c) (5%) Find an orthogonal matrix B that diagonalizes A .
- (d) (4%) Find the inverse matrix of B obtained in (c).
- (e) (4%) Find A^5 .

2. Let

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 2 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

- (a) (10%) Find the least squares solution of the system $Ax = b$
- (b) (5%) Find orthogonal projection of b on the column space of A

3. Let

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

- (a) (5%) Find the rank of A .
- (b) (5%) Find the nullspace of A .

4. (10%) Random variable X is uniformly distributed in $(-3,4)$, Y is uniformly distributed in $(5,7)$.
Let $D=X+2Y$, please find
 - (a) (5%) the probability density function (pdf) of D and
 - (b) (5%) the expectation of D , $E[D]$.
5. (10%) An urn contains 4 black balls and 6 white balls. A and B take turns to draw 1 ball randomly from the urn. A goes first, then B . The first person who draws a white ball wins. Please calculate the probability $P[B \text{ wins}]$ if

- (a) (5%) They replace the ball they drew.
(b) (5%) They do not replace the ball they drew.
6. (10%) The joint probability density function (pdf) of X and Y is defined as
- $$f_{X,Y}(x,y) = \begin{cases} c(x+y), & 3 \leq x \leq 5, \quad -2 \leq y \leq 4 \\ 0, & \text{otherwise} \end{cases}$$
- (a) (5%) Please find the marginal probability density function of X. (You will need to find out c=? first)
- (b) (5%) Are X and Y independent? Please verify your answer. (If you do not provide any verification, you will not get credit for this question.)
7. (10%) The probability that Taiwan has an earthquake over Richter magnitude scale 4 in a year is distributed as a Poisson(2) random variables. Please find that
- (a) (5%) The probability that there will be only 5 earthquakes over Richter magnitude scale 4 in the next 5 years?
- (b) (5%) The average number of earthquakes over Richter magnitude scale 4 in the next 5 years?
8. (10%) If the normal male adult's weight is distributed as a normal random variable with mean=65 and variance=25. Let us randomly pick a male adult and his weight is X, please find $P(70 < X \leq 90)$ and use standard normal cumulated distribution function $\Psi()$ to represent your answer. $\Psi(a)$ is

defined as $\int_{-\infty}^a \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx$