逢甲大學104學年度碩士班考試入學試題

編號：014　科目代碼：208

科目	微積分	適用系所	統計學系統計與精算碩士班應用統計暨計量財務組、精算組	時間	100 分鐘

※請務必在答案卷作答區內作答。　　共 一 頁第 一 頁

1) (10%) Prove that $\lim_{x\to 2} 2x - 1 = 3$ by $\varepsilon - \delta$ argument.

2) (15%) Evaluate the following limits.

(a) $\lim_{x\to 2} \frac{x-2}{x^2-4x+4}$ (5%)

(b) $\lim_{x\to 0} x \cot 3x$ (5%)

(c) $\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\pi - 2x}$ (5%)

3) (10%) Let $f(x) = x^4 - 2x^2 - 3x + 2$. Show that f(x) has exactly one critical number in the interval (1, 2).

4) (15%) Maximize and minimize $f(x, y) = xy$ on the unit circle $x^2 + y^2 = 1$.

5) (10%)

(a) State the fundamental theorem of calculus (5%)

(b) Let f be a continuous function that satisfies $f(x) = \int_0^x f(t)dt, \forall x \in R$.

Show that $f(x) = 0, \forall x \in R$. Hint : Take the derivative on the equation. (5%)

6) (10%) Show that the improper integral $\int_0^\infty e^{-x^2} dx$ has a value of $\frac{\sqrt{\pi}}{2}$.

7) (15%) If $f(x, y) = \frac{xy}{(x^2+y^2)^2}, (x, y) \neq (0,0)$, calculate the second order partial derivative $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = ?$

8) (15%) Find the volume formed by the intersection of two cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$.