逢甲大學104學年度碩士班考試入學試題

編號：003　科目代碼：302

科目	作業研究	適用系所	工業工程與系統管理學系B組	時間	100 分鐘

※請務必在答案卷作答區內作答。　

1. The ABC Insurance Company is introducing three new product lines: special risk insurance, mortgages and car loans. The expected profit is $2 per unit on special risk insurance, $4 per unit on mortgages and $3 per unit on car loans. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

	Work-Hours per Unit			Work-Hours
Department	Special Risk	Mortgage	Car Loans	Available
Underwriting	3	4	2	60
Administration	2	1	2	40
Claims	1	3	2	80

(a) Formulate a linear programming model for this problem.(10pt)

Let x4, x5 and x6 denote the slack variables for the respective constraints. After you apply the simplex method, a portion of the final simplex tableau is as follows:

Basic Variable	Eq.	Coefficient of:							RHS
		Z	x_1	x_2	x_3	x_4	x_5	x_6	
Z	(0)	1		0	0	5/6	2/3	0	
x_2	(1)	0		1	0	1/3	-1/3	0	
x_3	(2)	0	5/6	0	1	-1/6	2/3	0	50/3
x_6	(3)	0	-5/3	0	0	-2/3	-1/3	1	80/3

(b) Use the fundamental insight (revised simplex method) to identify the missing numbers in the final simplex tableau. Show your calculations(12pt)

(c) Supposed y1=1, y2=0, y3=0; use the complementary solutions property between primal and dual problems to find the following values (x1= , x2= , x3= , x4=0, x5= , x6=) and (y1=1, y2=0, y3=0, y4= , y5= , y6=) (8pt)

2. Consider the following problem
Maximize $Z = 2x_1 + 7x_2 - 3x_3$
Subject to
$x_1 + 3x_2 + 4x_3 \leq 30$
$x_1 + 4x_2 - x_3 \leq 10$
$x_1, x_2, x_3 \geq 0$
By letting x_4 and x_5 be the slack variables for the respective constraints, the simplex method yields the following final set of equations:
(0) $Z + x_2 + x_3 + 2x_5 = 20$
(1) $-x_2 + 5x_3 + x_4 - x_5 = 20$
(2) $x_1 + 4x_2 - x_3 + x_5 = 10$
Now you are to conduct sensitivity analysis by independently investigating each of the following two changes in the original model.

(a) Introduce a new variable x_6 with coefficients.
$\begin{bmatrix} c_6 \\ a_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ 2 \end{bmatrix}$, revise final tableau and reoptimize to find a new optimal solution if necessary.(10pt)

(b) Change the coefficients of x_1 to
$\begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, revise final tableau and reoptimize to find a new optimal solution if necessary.(10pt)

3. A factory has a single machining center on its shop floor. Jobs arrive at this machining center according to a Poisson process at a mean rate of 2 per day. The processing time to perform each job has an exponential distribution with a mean of 1/3 day. Because the jobs are bulky, those not being worked on are currently being stored in a room some distance from the machining center. However, to save time in fetching the jobs, the production manager is planning to add enough in-process storage space next to the machining center to accommodate 2 jobs in addition to the one being processed. (Excess jobs will continue to be stored temporarily in the distant room). Under this proposal, please determine

(a) The utilization for the machining center. (5pt)
(b) Average waiting time in storage for each job. (10pt)
(c) The probability that this storage space next to the machining center is not enough to accommodate all waiting jobs? (10pt)

4. In Taichung, iBike has become a new transportation means for citizens to bridge their trips to and from the public transportation system such as bus, BRT or train. Suppose *t(i,j)* represents the time to ride a bike from station *i* to station *j*. Let *t(u,v)* represent the time to walk from a location *u* to another location *v*. So, we can construct a biking network G=(N,A), where N denotes the set of locations (including stations), and A denotes the set of direct routes between any 2 locations.

(a) If there are *n* stations, how many direct biking routes are there between any 2 stations?(4pt)
(b) Suppose Tom plans to use iBike to move from location *s* to location *t*. He has to walk to a nearby iBike station *i* to rent a bike, ride it, return it to another iBike station *j* near his destination *t*, and then walk to his destination *t*. Please write down the total travel time for this *s-i-j-t* trip by using *t(?,?)*.(6pt)
(c) Let a binary integer *x(i,j)* represent whether Tom makes a direct move between locations (may be a location or a station) *i* and *j*. Write down the total travel time for Tom's *s-i-j-t* trip by using *t(?,?)* and *x(?,?)*.(6pt)
(d) To calculate the quickest path from s to t for Tom, we can use an integer linear program. In particular, the flow balance constraint indicates that a person (1) leaves his origin *s*; (2) enters and leaves an intermediate station *i* and *j* once, and then (3) arrives at his destination *t*. Please write down the 3 sets of flow balance constraints for this integer linear program. (9pt)