

中原大學 104 學年度碩士班考試入學

104/3/4 8:00 AM-9:30 AM

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

電機工程學系訊號與系統組

科目：工程數學(主考範圍：線性代數、微分方程) (共 1 頁，第 1 頁)

可使用計算機(僅限於四則運算、三角函數及對數等基本功能，可程式之功能不可使用)

不可使用計算機

In all of the following problems, derivations y, y' and y'' are all done with respect to the variable x . \mathbf{R} is the set of real numbers, and “ \times ” is the normal multiply operator in \mathbf{R} .

1. (10%) Solve the following initial value problem:

$$y' = x^3 y, \quad y(0) = 2$$

2. (10%) Find general solution of the following ODE:

$$y' - \frac{1}{x}y = x^2$$

3. Find general solution of the following ODE:

(a) (10%) $y'' + 2y' - 3y = 0$

(b) (10%) $y'' + 2y' - 3y = 8e^x$

4. (10%) Find general solution of the following ODE:

$$y'' + 2y' + 4y = 0$$

5. (a) (10%) Let S be a subset of \mathbf{R}^2 , $S = \{(a + b, a \times b) \mid a, b \in \mathbf{R}\}$.

Is S a subspace of \mathbf{R}^2 ? Explain your answer, and give its dimension if yes.

- (b) (10%) Let W be a subset of \mathbf{R}^3 , $W = \{(a + b + c, 2a + b + 2c, 4a + 3b + 4c) \mid a, b, c \in \mathbf{R}\}$. Is W a subspace of \mathbf{R}^3 ? Explain your answer, and give its dimension if yes?

6. (a) (5%) What is a linear transformation?

(b) (10%) Let T be a transformation from \mathbf{R}^3 into \mathbf{R}^3 . $T(a, b, c) = (a - b, b - c, c - a)$, where $a, b, c \in \mathbf{R}$. Is T a linear transformation? If yes, then also find the kernel and range of T .

7. Let $M = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$.

(a) (10%) Find matrix P , and D such that $M = PDP^{-1}$, where D is a diagonal matrix.

(b) (5%) Find $\lim_{n \rightarrow \infty} M^n$.