

中原大學 104 學年度碩士班考試入學

104/3/4 8:00 AM-9:30 AM

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

機械工程學系丙組

科目：工程數學【分工程數學(A)、工程數學(B)兩部份計分，各佔 50 分】(共 1 頁，第 1 頁)

- 可使用計算機(僅限於四則運算、三角函數及對數等基本功能，可程式之功能不可使用)
 不可使用計算機

第一部份：工程數學(A) 50%

1. (15%) Solve the given nonlinear first-order differential equation.

$$\frac{dy}{dx} = \frac{y^2 + xy^3}{-5y^2 + xy - y^3 \sin y}$$

2. (15%) Find the general solution of the given differential equation.

$$y'' + 2y' + y = e^{-x} \ln x.$$

3. (20%) Solve the given initial-value problem.

$$y''' - 2y'' + y' = 2 - 24e^x + 20e^{3x}, y(0) = \frac{1}{2}, y'(0) = \frac{5}{2}, y''(0) = -\frac{9}{2}.$$

第二部份：工程數學(B) 50%

4. (15%) Use the Laplace transform to solve the given system of differential equations.

$$\begin{cases} y_1'' + y_1' + y_2' = 0 \\ y_2'' + y_2' - 4y_1' = 0 \end{cases}, y_1(0) = 1, y_1'(0) = 0, y_2(0) = -1, y_2'(0) = 5.$$

5. (15%) Evaluate the line integral $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$, where $\vec{F} = 2z\vec{i} + 2x\vec{j} + y\vec{k}$ and C is the helix from $A: (1, 0, 0)$ to $B: (1, 0, 4\pi)$ as shown in Fig. 1.

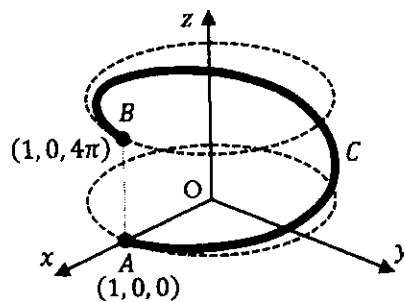


Fig. 1

6. (20%) Find the Fourier series of the given $f(x)$ as shown in Fig. 2, which is assumed to have the period 2π .

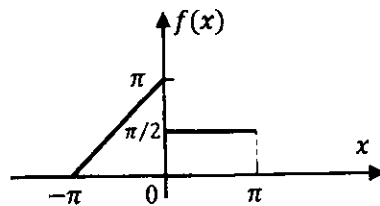


Fig. 2