中原大學 104 學年度碩士班考試入學

104/3/4 10:10 AM~11:40 AM

誠實是我們珍視的美德, 我們喜愛「拒絕作弊,堅守正直」的你!

(15%)

應用數學系資訊科學組

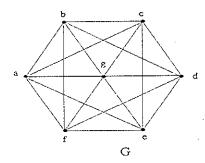
科目: 離散數學

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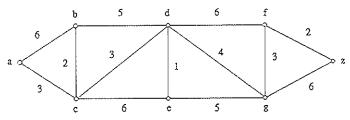
□可使用計算機(僅限於四則運算、三角函數及對數等基本功能,可程式之功能不可使用)

■不可使用計算機

- 1. Show that if f and g are real-valued functions such that f(x) is O(g(x)), then $f^2(x)$ is $O(g^2(x))$. [Note that $f^2(x) = [f(x)]^2$.]. (10%)
- 2. Show that an undirected graph has an even number of vertices of odd degree.(10%)
- 3. Devise a recursive algorithm to find the nth term of the sequence defined by $a_0=1$, $a_1=1$, and $a_n=a_{n-1}+2a_{n-2}$, for $n=2,3,4,\cdots$. (10%)
- 4. Use Kuratowski's Theorem to show that the graph G is nonplanar. (10%)



5. Find a shortest path between a and z in the following weighted graphs.



6. Find the number of integer solutions to the equation

$$x_1 + x_2 + 3x_3 = 24$$
,
where $x_1 \ge 1, x_2 \ge 0$ and $x_3 \ge 2$. (15%)

- 7. Find a general solution to $a_n 7a_{n-1} + 10a_{n-2} = 3.$ (15%)
- 8. Find all solutions to the system of congruences. (15%)

$$x \equiv 2 \pmod{3}$$

$$x \equiv 1 \pmod{7}$$

$$x \equiv 3 \pmod{13}$$