

中原大學 104 學年度碩士班考試入學

104/3/4 10:10 AM~11:40 AM

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

應用數學系統計組：應用數學系統計組在職生(在職)

科目： 統計

(共 1 頁，第 1 頁)

可使用計算機(僅限於四則運算、三角函數及對數等基本功能，可程式之功能不可使用)

不可使用計算機

1. Give the definitions for the following terms :

- (a) Maximum likelihood estimator
- (b) Type I error
- (c) Type II error
- (d) Power function
- (e) p-value
- (f) Sufficient statistic (30%)

2. Let X_1, X_2, \dots, X_n be a random sample from $f(x : \theta)$,
where

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} & 0 < x \leq \theta \\ 0 & \text{o.w} \end{cases}, \quad 0 < \theta < \infty$$

Find the maximum likelihood estimator $\hat{\theta}$ of θ . (15%)

3. Let X_1, X_2, \dots, X_n denote a random sample from a distribution that is Normal distribution $N(\theta_1, \theta_2)$, $-\infty < \theta_1 < \infty$, $0 < \theta_2 < \infty$.

- (a) Find the maximum likelihood estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ of θ_1 and θ_2 .
- (b) Find a sufficient statistic for (θ_1, θ_2) .
- (c) Find an unbiased estimator for θ_2 based on X_1, X_2, \dots, X_n . (45%)

4. Let the joint p.d.f. of X and Y be $f(x, y) = Ce^{-(x^2+2xy+2y^2)}$, $-\infty < x < \infty$, $-\infty < y < \infty$.

- (a) Find C . (4%)
- (b) Find $E(X)$, $E(Y)$, $\text{Var}(X)$, $\text{Var}(Y)$, and the correlation coefficient of X and Y . (3%)
- (c) Find $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy e^{-(x^2+2xy+2y^2)} dx dy$. (3%)