中原大學104學年度碩士班考試入學

104/3/4　8:00 AM~9:30 AM

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

應用數學系數學組；應用數學系數學組在職生(在職)

科目：高等微積分　　　　(共1頁，第1頁)

☐可使用計算機(僅限於四則運算、三角函數及對數等基本功能，可程式之功能不可使用)
■不可使用計算機

-------------------------(不可直接作答於試題，請作答於答案卷)-------------------------

一、　True or False(each 3 points)

(a) The intersection of any open sets is open.

(b) The interval (0,1) is open relative to a subset in the Euclidean 2-space.

(c) Every derivative function has the intermediate value property.

(d) The inverse of a one-one , onto and continuous function must be also continuous.

(e) Every bounded infinite subset in the Euclidean n-space has a limit point there.

(f) Every bounded sequence in a metric space contains a convergent subsequence.

(g) A set E is dense in a metric space X if every point of X is a limit point of E.

(h) The limit function of a uniformly convergent sequence of continuous funnctions on a closed interval must be Riemann-integrable.

(i) The product of any two convergent sequences must converge.

(j) A continuous function on a compact metric space must be uniformly continuous.

(k) Every monotonic function must be Riemann-integrable on a closed interval.

(l) Every continuous mapping preserves the compactness and connectness.

二、　Explain the following terminologies(each 4 points):

(a) Compactness　(b)uniform continuity　(c)Connectness　(d)Cauchy Sequence

三、　Prove the following statements(each 6 points):

(a) Every neighborhood in a metric space is an open set.

(b) Let E be the set of all numbers $1/n$, where $n = 1,2,3,\ldots$. Then $\sup E = 1$.

(c) Let $E = \{x \in X : f(x) = 0\}$, where f is a real continuous function on a metric space X, then E is closed.

(d) The set $\{1/n : n = 1,2,3\ldots\}$ has a unique limit point in the Euclidean 1- space.

(e) Convergent sequence in a metric space must be bounded.

(f) Every sequence in a compact metric space has a convergent subsequence.

(g) Consider the series $1/2 + 1/3 + 1/2^2 + 1/3^2 + 1/2^3 + 1/3^3 + \cdots$. Then the root test indicates convergence while the ratio test does not apply.

(h) Let $f_n(x) = n^2 x(1-x^2)^n, x \in [0,1]$, then $\{f_n\}$ does not converge uniformly on $[0,1]$.