

考生作答須知：

- 一、本試卷共有兩部分，各佔 50 分。每題配分標示於題後，總分 100 分。
- 二、答案請於答案卷上依題號次序作答，題號務必標示清楚，並寫出計算過程，否則不予計分。

第一部分：微分 (50%)

1. $\lim_{x \rightarrow 1} \frac{\ln(\cos(x-1))}{1 - \sin(\frac{\pi}{2}x)}$ (5%)
2. $\lim_{x \rightarrow 0} \frac{\sqrt{1 + \tan x} - \sqrt{1 + \sin x}}{x^3}$ (5%)
3. $y = \frac{1 + \sin x}{\sqrt{x}}$ ， $x^2 y'' + xy' + (x^2 - \frac{1}{4})y = ?$ (5%)
4. $f(x) = (\sqrt{x+2})^{\sqrt{4x}}$ ， $f'(4) = ?$ (5%)
5. $f(x) = \frac{x-5}{x^2-x-2}$ ， $\frac{f^{(100)}(0)}{100!} = ?$ (10%)
6. 利用一階導數極值判斷法，證明 $\pi^e < e^\pi$ 。(10%)
7. $f(x) = x^5 - 5x^3 + 10x - 3$ ，求相對極大值及相對極小值 (10%)

第二部分：積分 (50%)

8. A manufacturer of CCU-Pads, tablet PC, finds that its total cost in producing 100 CCU-Pads per week is $C(100) = \$2150$ and that its marginal cost is $MC(x) = 10 + 0.2x$ dollars per CCU-Pad at production level x CCU-Pads per week.
 - (1) Find the amount by which total cost will increase if production is increased from $a=100$ to $b=120$ CCU-Pads per week. (5%)
 - (2) Find the total cost in producing 120 CCU-Pads per week. (5%)
9. Find
 - (1) $\int_1^e \sqrt{x} \ln x dx$ (5%)
 - (2) $\int e^x \cos x dx$ (5%)

10. Evaluate $\int \sec 2x dx$ and $\int \sec^2 2x dx$ (10%)

11. Prove $\int_1^2 dx \int_0^2 (x^2 + xy + y^2) dy = \int_0^2 dy \int_1^2 (x^2 + xy + y^2) dx$ (10%)

12. Evaluate the iterated integral $\int_0^4 \left\{ \int_1^{\sqrt{y}} (x + y) dx \right\} dy$ (10%)