

# 淡江大學 104 學年度碩士班招生考試試題 15

系別：數學學系 B 組

科目：基礎數學（含微積分、線性代數）

考試日期：3 月 8 日（星期日）第 2 節

本試題共 10 大題， 1 頁

請詳列計算過程，否則不予計分，每題 10 分，共 100 分

1. (a)  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$       (b)  $\frac{d}{dx} \left[ \int_x^4 \tan^2 u \cos u du \right]$
2. (a)  $\int_1^2 \frac{dx}{(3-5x)^2}$       (b) Find  $\frac{dy}{dx}$ , if  $y = \sin(x^2)$
3.  $\int t^2 e^t dt$ .
4.  $\int_0^1 \int_y^1 e^{x^2} dx dy$ .
5. Use Lagrange multipliers to find the maximum and minimum values of the function  $f(x, y, z) = 2x + 2y + z$  subject to the given constraint  $x^2 + y^2 + z^2 = 9$ .
6. Use Gauss-Jordan elimination method to solve the following system of linear equations
$$\begin{aligned}x + y + 2z &= 9 \\2x + 4y - 3z &= 1 \\3x + 6y - 5z &= 0\end{aligned}$$
7.  $A$  is a  $4 \times 4$  matrix and its determinant is  $\det(A) = -2$ ,  
find  $\det(-A)$ ,  $\det(A^{-1})$ ,  $\det(2A^T)$ ,  $\det(A^3)$ , and  $\det((3A)^{-1})$ .
8. (a) Show that the vectors  $\mathbf{v}_1 = (1, 2, 1)$ ,  $\mathbf{v}_2 = (2, 9, 0)$ , and  $\mathbf{v}_3 = (3, 3, 4)$  form a basis for  $R^3$ .  
(b) From 8 (a), find the coordinate vector of  $\mathbf{v} = (5, -1, 9)$  relative to the basis  $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ .
9. Find the all eigenvalues and bases for the eigenspaces of the matrix  $\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$ .
10. Find the rank and nullity of the following matrix
$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$