東吳大學 104 學年度碩士班研究生招生考試試題

系級	企業管理學系碩士班 C 組	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1. At a certain factory, output is given by $Q = 60K^{1/3}L^{2/3}$ units, where K is the capital investment (in thousands of dollars) and L is the size of the labor force, measured in worker-hours. If output is kept constant, at what rate is capital investment changing at a time when $K = 8$, $L = 1{,}000$, and L is increasing at the rate of 25 worker-hours per week? (10 marks)
2. Sketch the graph of the given function $f(x) = 1/(x^2 - 9)$. (10 marks)
3. Suppose the demand for a certain commodity is given by $q = b - ap$, where a and b are positive constants, and $0 \le p \le b/a$.
   a. Express elasticity of demand as a function of p. (5 marks)
   b. Show that the demand is of unit elasticity at the midpoint $p = b/(2a)$ of the interval $0 \le p \le b/a$. (5 marks)
   c. For what values of p is the demand elastic? Inelastic? (5 marks)
4. Paula Perkins, the owner of Paula's perfume Shoppe, expects to sell 800 bottles of a certain brand of perfume this year. The perfume costs \$20 per bottle, the ordering fee is \$10 per shipment, and the cost of storing the perfume is 40 cents per bottle per year. The perfume is sold at a constant rate throughout the year, and each shipment arrives just as the preceding shipment is being used up.
   a. How many bottles should Paula order in each shipment to minimize total cost? (10 marks)
   b. How often should Paula order the perfume? (5 marks)
5. A manufacturer of machinery parts determines that q units of a particular piece will be sold when the price is $p = 110 - q$ dollars per unit. The total cost of producing those q units is $C(q)$ dollars, where $C(q) = q^3 - 25q^2 + 2q + 3{,}000$.
   a. For what value of q is profit maximized. (10 marks)
   b. Find the consumer's surplus when profit is maximized. (10 marks)
6. Find the given integral. $\int \frac{\ln x}{x^2}\,dx$. (10 marks)
7. Suppose that you wish to construct a rectangular box with a volume of 32 ft^3. Three different materials will be used in the construction. The material for the sides costs \$1 per square foot, the material for the bottom costs \$3 per square foot, and the material for the top costs \$5 per square foot. What are the dimensions of the least expensive such box? (10 marks)
8. A consumer has \$280 to spend on two commodities, the first of which costs \$2 per unit and the second \$5 per unit. Suppose that the utility derived by the consumer from x units of the first commodity and y units of the second is given by $U(x, y) = 100x^{0.25}y^{0.75}$. How many units of each commodity should the consumer buy to maximize utility? (10 marks)