國立臺南大學104學年度 **資訊工學系碩士班** 招生考試 離散數學與線性代數 試題卷

一、離散數學 (50%)

1. (8%) For the variables x and y in the universe comprises all nonzero integers. Determine the statements in the following are truth or false. (2% for each)
   (A) $\exists x\, \forall y\, [xy = 1]$
   (B) $\forall x\, \exists y\, [2x + y = 5]$
   (C) $\forall x\, \exists y\, [x - 3y = -8]$
   (D) $\exists x\, \exists y\, [(2x + y = 5) \wedge (x - 3y = -8)]$

2. (6%) What is the positive integer n that the following equations
   $$(1)\quad x_1 + x_2 + x_3 + \cdots + x_{20} = n,$$
   $$(2)\quad y_1 + y_2 + y_3 + \cdots + y_{60} = n$$
   have the same number of positive integer solutions?

3. (6%) If $a, b, c \in \mathbf{Z}$ and $31|(5a + 7b + 11c)$, prove that $31|(21a + 17b + 9c)$.

4. Given a set $S = \{2, 3, 5, 7, 21, 42, 105, 210\}$, a relation **R** on $S \times S$ is defined as "x divides y", where $x, y \in S$.
   (a) (6%) Show that the relation **R** is partial ordering.
   (b) (4%) Draw the Hasse diagram for the relation **R**.
   (c) (4%) List the maximal elements and minimal elements in **R** if they exist.
   (d) (4%) List the greatest element and the least element in **R** if they exist.

5. (6%) How many positive divisors are there for 20077200.

6. (6%) For every positive odd integer n, prove the following equation by induction
   $$(-2)^0 + (-2)^1 + (-2)^2 + \cdots + (-2)^n = \frac{1 - 2^{n+1}}{3}.$$

二、線性代數 (50%)

1. (15%) Solve the following system by Gauss-Jordan elimination.

$$\begin{aligned} x_1 + 2x_2 - x_3 + 3x_4 &= 4 \\ 2x_1 + 4x_2 - 2x_3 + 7x_4 &= 10 \\ -x_1 - 2x_2 + x_3 - 4x_4 &= -6 \end{aligned}$$

2. (15%) Find the eigenvalue(s) and the corresponding eigenvectors of the following matrix : $A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3. (10%) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, assuming that $\det(A) = -2$, find the determinants in

(a) $\begin{vmatrix} a-g & b-h & c-i \\ g & h & i \\ d & e & f \end{vmatrix}$ (b) $\begin{vmatrix} -c & 2b & a \\ -i & 2h & g \\ -f & 2e & d \end{vmatrix}$

4. (10%) Find a basis for the column space of the matrix $A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$