1. Consider the basis $S = \{\mathbf{v}_1, \mathbf{v}_2\}$ for $\mathbb{R}^2$, where $\mathbf{v}_1 = (1,0)$ and $\mathbf{v}_2 = (1,1)$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear operator such that
$$T(\mathbf{v}_1) = (-1,2) \quad \text{and} \quad T(\mathbf{v}_2) = (2,-3).$$
Find a formula for $T(x_1, x_2)$, and use that formula to find $T(5,-3)$. (20%)

2. Let P_2 be the vector space of all polynomials of degree at most 2 and $T : P_2 \to P_2$ be defined by
$$T(a_0 + a_1 x + a_2 x^2) = (2a_0 - a_1 + 3a_2) + (4a_0 - 5a_1)x + (a_1 + 2a_2)x^2.$$
Find the eigenvalues and the corresponding eigenvectors of T. (20%)

3. Find the determinant of the matrix $B = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 2 & -1 & 1 \\ 2 & 1 & 3 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ (20%)

4、(1) 何謂 reduced row-echelon form (簡化階梯形矩陣)？(10%)

   (2) 求出下列矩陣的 reduced row-echelon form。(10%)

$$\begin{bmatrix} 1 & 2 & -1 & -1 \\ 2 & 4 & -1 & 4 \\ -1 & -2 & 3 & 5 \end{bmatrix}$$

5、State the definition of simple groups, and give a nontrivial example. (15%)

6、(1) 何謂有限環 (finite ring)？(8%)

   (2) 舉出一個不是有限域 (finite field) 的有限環實例。(7%)