

1. (10%) (A) Describe definition of continuity of a function.

(10%) (B) For what values of a and b is $f(x) = \begin{cases} ax + 2b & \text{if } x \leq 0 \\ x^2 + ax - b & \text{if } 0 < x \leq 2, \\ 3x - 1 & \text{if } 2 < x \end{cases}$

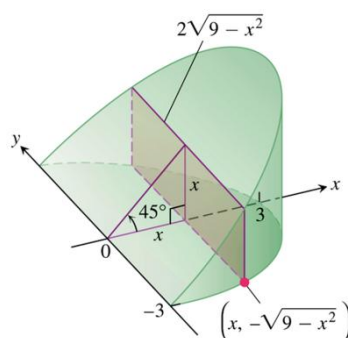
continuous at every x .

2. (10%) (A) Describe the Mean Value Theorem.

(10%) (B) Use the Mean Value Theorem to prove the inequality

$|\sin a - \sin b| \leq |a - b|$ for all a and b .

3. (10%) A curved wedge is cut from a circular cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the center of the cylinder, see the following figure. Find the volume of the wedge.



4. (10%) (A) Describe the Green's Theorem.

(10%) (B) Evaluate the integral $\int_C y^3 dx - x^3 dy$, where C is the circle

$$x^2 + y^2 = 4.$$

5. (10%) Prove that

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \text{ for } x \in [-1, 1],$$

where $\tan^{-1} x$ is the arc tangent function.

6. (10%) Determine whether the function

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is a rational number,} \\ 0, & \text{if } x \text{ is an irrational number} \end{cases}$$

is continuous on \mathbb{R} .

7. (10%) Determine whether the function $(\ln x)^{1-x}$ is uniformly continuous on $(1, 2)$.

