國立高雄大學 104 學年度研究所碩士班招生考試試題

科目：統計學　　系所：金融管理學系　　是否使用計算機：是
考試時間：100 分鐘　　本科原始成績：100 分

1. Answer the following questions:

(1) (5%) If A and B are independent events with P(A) = 0.65 and P(A ∩ B)= 0.26, then, P(B) = ?

(2) (5%) If P(A) = 0.4, P(B| A) = 0.35 and P(A ∪ B) =0.69, then P(B) = ?

2. (5%) Prove $\frac{\sum(x_i-\bar{x})^2}{n-1}=\frac{\sum x_i^2-n\bar{x}^2}{n-1}$, where x_i is the value of random variable, n is the sample number, and $\bar{x}=\frac{\sum x_i}{n}$.

3. (5%) What is the moment generating function if a random variable X has Poisson distribution with parameter λ.

4. Suppose you are asked to estimate a linear regression model: $y_i = b_0 + b_1 x_i + \varepsilon_i$.

Based on the following data:

x_i	4	3	5	2	8
y_i	15	15	10	17	6

Answer the following questions:

(1) (10%) What are the estimated values of b_0 and b_1?

(2) (5%) Use the t -test to test the following hypotheses ($\alpha = 0.05$): $H_0 : b_1 = 0$

(3) (5%) Compute the coefficient of determination R^2.

5. Given a multivariate linear model as follows: $Y = X\beta + \varepsilon$, where Y and ε are $n\times1$ vectors respectively, β is a $(k+1)\times1$ vector, and X is a $n\times(k+1)$ matrix. n and k are constant values. Please show that

(1) (5%) $\hat{\beta}=(X'X)^{-1}X'Y$, where $\hat{\beta}$ is the estimator for β.

(2) (5%) $Var(\hat{\beta})=\sigma^2(X'X)^{-1}$.

6. (5%) We select *n* observations $X_1, X_2, X_3 \ldots\ldots, X_n$ from a population. Assume X~N(μ, σ^2), please tell me which equation (E_1, E_2, or E_3) in the following is a better estimator for μ?

   (a) $E_1 = X_2$

   (b) $E_2 = \frac{X_1}{2} + \frac{1}{2(n-1)}(X_2 + X_3 + \cdots + X_n)$

   (c) $E_3 = X_1 + X_2 + X_3$

7. What is known as the characteristics line of modern investment analysis is simply the regression line obtained from the following model:

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_t$$

where r_{it}=the rate of return on the i^{th} security at time t
r_{mt}=the rate of return on the market portfolio at time t
ε_t=stochastic disturbance term

In this model β_i is known as the beta coefficient of the i^{th} security, a measure of market (or systematic) risk of a security. One the basis of 240 monthly rates of return for the period 2000-2014, we obtained the following characteristic line for IBM stock in relation to the market portfolio index.

$$\widehat{r_{it}} = 0.7264 + 1.0598 r_{mt}$$
$$\text{se} = (0.3001)\ \ (0.0728)$$
$$R^2 = 0.4710$$

Please answer the following questions using $\alpha = 5\%$.

(1) (5%) What is the degree of freedom of this equation?

(2) (5%) A security whose beta coefficient is greater than 1 is said to be a volatile security. Was IBM a volatile security in the time period?

(3) (5%) Is the intercept coefficient significantly different from zero? If it is (or is not), what is its practical meaning related to the efficient market hypothesis (EMH)?

8. Hereunder is a debate between Harry Porter (哈利波特), Ron Wesley (榮恩衛斯里), and Hermione Granger (妙麗格蘭傑) in the Gryffindor house (葛萊芬多學院). Please use the data provided to justify which argument is right. Please note three issues: (1) the variance of population is unkown; (2) $\alpha = 5\%$; (3) always use a two-tailed hypothesis test.

Ron: Gee….Oh my God, I fail the Potions course (魔藥學)!!! No…I don't want to see Professor Snape (石內卜) again in the next semester. This is unfair. Hermione always passes all courses.

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Girls are smarter than boys because they always get higher scores in exams. We boys should have easier questions in exams.

Harry: Mmm….I don't think girls are smarter than boys. See, I am the seeker in Quidditch (魁地奇，一種運動). I need to practice Quidditch every day but I still pass the Potions course. I guess you should study harder to pass the exams.

(1) (5%) Do girls always get higher scores in exams? We randomly select 30 students to be a sample. Please use the data provided below and use a proper statistical method to tell me whether Ron's argument is true. Please write down your hypotheses (null and alternative) and the empirical results.

Then, Hermione shows up. After listening their debate, she argues.

Hermione: No, girls are not smarter than boys. You slacker! The reason we get higher scores is that we study harder. We spend more time on studying!

(2) (5%) Please examine whether girls spend more time on studying every week by using a proper statistical method. Please write down your hypotheses (null and alternative) and the empirical results.

Ron: Okay, okay, you are right. I should study harder….Hey, we should analyze whether studying harder really can get higher scores.

(3) (5%) Please use one kind of bivariate descriptive statistics to show whether scores and studying time are related. Write down the empirical results.

(4) (5%) Please build a regression model to examine whether studying harder can get higher scores. Please show me the equation, describe the meaning of your variables, and tell me your expected results (expected result：期望的結果而非實際結果) (Note: You do NOT need to compute the results).

Hermione: Wait, don't you think Professor Snape favor the students in the Slytherin house (史萊哲林學院)? The students from Slytherin house usually get higher scores in the Potions course.

Harry: Agree!

Ron: Agree!

(5) (5%) Is Professor Snape biased? Does he really prefer the students in the Slytherin house? Please use a proper statistical method to investigate whether the students in the four houses get significantly different scores. Write down your hypotheses (null and alternative) and the empirical results.

(6) (5%) Please build one regression equation to examine Professor Snape is biased. Please show me the equation, describe the meaning of your variables, and tell me your expected results (Note: You do NOT need to compute the results).

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Sex (B=boy, G=girl)	Exam score	The hours of studying time per week	House (G=Gryffindor, S=Slytherin, R=Ravenclaw, H=Hufflepuff)
B	46	10	G
B	96	22	G
B	39	9	G
B	89	21	G
B	70	16	G
B	69	16	G
G	75	17	G
G	100	23	G
B	46	10	H
B	37	8	H
G	92	21	H
G	49	11	H
G	77	18	H
G	86	20	H
G	59	13	H
G	68	16	H
B	22	5	R
B	70	16	R
B	60	14	R
B	44	10	R
B	56	13	R
G	87	20	R
G	78	18	R
B	85	20	S
B	57	13	S
B	95	22	S
G	92	21	S
G	72	17	S
G	94	22	S
G	68	16	S

Table 2: t-Distribution Critical Values

The entries in the table below are the critical values $t_{n,p}$, where n represents the number of degrees of freedom and p is the upper tail probability. That is, if T has the t-distribution with n degrees of freedom, then the value in the table represents the number $t_{n,p}$ such that $P\left(T > t_{n,p}\right) = p$.

Upper Tail Probability p

d.f.	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	1.376	1.963	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725

26	**0.856**	**1.058**	**1.315**	**1.706**	**2.056**	**2.479**	**2.779**	**3.067**	**3.435**	**3.707**
27	**0.855**	**1.057**	**1.314**	**1.703**	**2.052**	**2.473**	**2.771**	**3.057**	**3.421**	**3.690**
28	**0.855**	**1.056**	**1.313**	**1.701**	**2.048**	**2.467**	**2.763**	**3.047**	**3.408**	**3.674**
29	**0.854**	**1.055**	**1.311**	**1.699**	**2.045**	**2.462**	**2.756**	**3.038**	**3.396**	**3.659**
30	**0.854**	**1.055**	**1.310**	**1.697**	**2.042**	**2.457**	**2.750**	**3.030**	**3.385**	**3.646**

Upper Tail Probability p

d.f.	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
35	**0.852**	**1.052**	**1.306**	**1.690**	**2.030**	**2.438**	**2.724**	**2.996**	**3.340**	**3.591**
40	**0.851**	**1.050**	**1.303**	**1.684**	**2.021**	**2.423**	**2.704**	**2.971**	**3.307**	**3.551**
45	**0.850**	**1.049**	**1.301**	**1.679**	**2.014**	**2.412**	**2.690**	**2.952**	**3.281**	**3.520**
50	**0.849**	**1.047**	**1.299**	**1.676**	**2.009**	**2.403**	**2.678**	**2.937**	**3.261**	**3.496**
55	**0.848**	**1.046**	**1.297**	**1.673**	**2.004**	**2.396**	**2.668**	**2.925**	**3.245**	**3.476**
60	**0.848**	**1.045**	**1.296**	**1.671**	**2.000**	**2.390**	**2.660**	**2.915**	**3.232**	**3.460**
65	**0.847**	**1.045**	**1.295**	**1.669**	**1.997**	**2.385**	**2.654**	**2.906**	**3.220**	**3.447**
70	**0.847**	**1.044**	**1.294**	**1.667**	**1.994**	**2.381**	**2.648**	**2.899**	**3.211**	**3.435**
75	**0.846**	**1.044**	**1.293**	**1.665**	**1.992**	**2.377**	**2.643**	**2.892**	**3.202**	**3.425**
80	**0.846**	**1.043**	**1.292**	**1.664**	**1.990**	**2.374**	**2.639**	**2.887**	**3.195**	**3.416**
85	**0.846**	**1.043**	**1.292**	**1.663**	**1.988**	**2.371**	**2.635**	**2.882**	**3.189**	**3.409**
90	**0.846**	**1.042**	**1.291**	**1.662**	**1.987**	**2.368**	**2.632**	**2.878**	**3.183**	**3.402**
95	**0.845**	**1.042**	**1.291**	**1.661**	**1.985**	**2.366**	**2.629**	**2.874**	**3.178**	**3.396**
100	**0.845**	**1.042**	**1.290**	**1.660**	**1.984**	**2.364**	**2.626**	**2.871**	**3.174**	**3.390**
150	**0.844**	**1.040**	**1.287**	**1.655**	**1.976**	**2.351**	**2.609**	**2.849**	**3.145**	**3.357**
250	**0.843**	**1.039**	**1.285**	**1.651**	**1.969**	**2.341**	**2.596**	**2.832**	**3.123**	**3.330**
1000	**0.842**	**1.037**	**1.282**	**1.646**	**1.962**	**2.330**	**2.581**	**2.813**	**3.098**	**3.300**