國立高雄大學 104 學年度研究所碩士班招生考試試題

科目：離散數學與資料結構　系所：資訊工程學系　是否使用計算機：否
考試時間：100 分鐘　本科原始成績：100 分

單選題（每題2分，共3題）

1. A(n) _____ is a tree in which each internal node has either two or three children, and all leaves are at the same level.

a) red-black tree
b) 2-3 tree
c) 2-3-4 tree
d) AVL tree

2. In an array based representation of a complete binary tree, which of the following represents the left child of node tree[i]?

e) tree[i+2]
f) tree[i–2]
g) tree[2*i+1]
h) tree[2*i+2]

3. The minimum height of a binary tree of *n* nodes is _____.

i) n
j) $n / 2$
k) $(n / 2) - 2$
l) $\lceil \log_2(n + 1) \rceil$

問答題

4. There are 9 students to be assigned into a number of unlabeled groups.
(a) (5%) In how many ways can they be assigned into 3 groups and each group has 3 students?
(b) (5%) In how many ways can they be assigned into 4 groups and each group has at most 3 students?

5. (a) (5%) Find a formula for $1\times3+2\times4+3\times5+\cdots+n\times(n+2)$, $n\in Z^+$.
(b) (5%) Use the Principle of Mathematical Induction to prove the formula in (a).

6. For all $n\in Z^+$, show that
(a) (5%) $\sum_{i=0}^{n}(-1)^i\binom{n}{n-i}(n-i)^n = n!$

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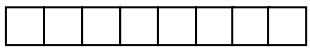
(b) (5%) $\sum_{i=0}^{n}(-1)^i\binom{n}{n-i}(n-i)^{n+1} = n!\times\binom{n+1}{2}$

(Hint: Use the Stirling numbers of the second kind.)

7. There are 6 vertices in an undirected graph. We want to add edges to connect these 6 vertices such that no vertex is isolated.
(a) (5%) In how many ways can 5 edges be added if each vertex is connected to at most 2 edges?
(b) (5%) In how many ways can 5 edges be added?

8. Suppose we have a 1×8 chessboard as shown in the following.
(a) (5%) In how many ways can we tile this 1×8 chessboard using 1×1 domino or 1×2 (horizontal) dominos?
(b) (5%) In how many ways can we tile this 1×8 chessboard using green or red 1×1 domino, or black or white 1×2 (horizontal) dominos?

Chessboard:  1×1 domino:  1×2 domino: 

9. (a) Derive a recursive formula for computing the binomial coefficient $\binom{n}{m}$,
where $\binom{n}{0} = \binom{n}{n} = 1$. (5%)
(b) Write a recursive function to compute the binomial coefficient. (8%)
(c) Analyze the time requirement of your algorithm. (5%)
(d) Analyze the space requirement of your algorithm. (5%)

10. Assuming C precedence, write the postfix form for the following expressions:
(a) A&&B||C||!(E>F)　　(2%)
(b) !(A&&!((B<C)||(C>D)))||(C<E)　　(3%)

11. Write a function ***length*** to count the number of nodes in a chain. What is the time complexity of your function. (5%)

12. Given a non-directed graph below, in which each number on the edges represents the edge cost, and each number in the circles represents the vertex index.

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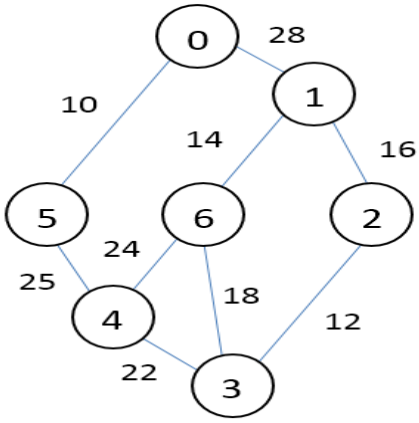

(a) Show how to use Kruskal's method to generate the minimum cost spanning tree. Show the running steps clearly. (4%)
(b) What is the corresponding minimum cost? (1%)

13. Input a sequence of data in the order from left to right:

82, 16, 9, 95, 27, 75, 42, 69, 34

Show how to construct the AVL tree step-by-step. (6%)