國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：應用數學【物理系碩士班】

題號：4045

1. Find the radial components of the velocity and acceleration v_ρ and a_ρ of a moving particle in circular cylindrical cooridantes.
Hint: $\mathbf{r}(t)=\hat{\rho}(t)\rho(t)+\hat{z}z(t)=[\hat{x}\cos\varphi(t)+\hat{y}\sin\varphi(t)]\rho(t)+\hat{z}z(t)$,
where $\hat{\rho}$ is the unit vector along the radial direction.(10%)

2. $\delta(g(x))$ is the delta function of a function $g(x)$ with simple zeros at $x=a_i$ in the real axis. Show that

$$\delta(g(x))=\sum_{\substack{a_i,\\ g(a_i)=0,\\ g'(a_i)\neq 0}}\frac{\delta(x-a_i)}{|g'(a_i)|}.(10\%)$$

3. Evaluate the function of matrix $\exp(i\sigma_x\varphi)=?$,
where i is the imaginary unit,
and $\sigma_x=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, one of the Pauli matrices.(10%)

4. A boat, coasting through the water, experiences a resisting force proportional to v^n, v being the boat's instantaneous velocity. Newton's second law leads to

$$m\frac{dv}{dt}=-kv^n.$$

With $v(t=0)=v_0, x(t=0)=0$, find v as a function of time and v as a function of distance.(10%)

5. A quantum mechanical analysis of the Stark effect leads to the differential equation

$$\frac{d}{d\xi}\left(\xi\frac{du}{d\xi}\right)+\left(\frac{1}{2}E\xi+\alpha-\frac{m^2}{4\xi}-\frac{1}{4}F\xi^2\right)u=0.$$

Here α is a separation constant, E is the total energy, and F is a constant. Using the larger root of the indical equation, develop a power-series solution about ξ=0. Evaluate the first three coefficients in terms of zero order coefficient a_0. Note that the constant F does not appear untial a_3 is included. (10%)

6. A triangular wave is represented by

$$f(x)=\begin{cases} x, & 0<x<\pi \\ -x, & -\pi<x<0. \end{cases}$$

Represent $f(x)$ by a Fourier series.(10%)

7. The function $f(x)=\begin{cases} 1, & |x|<1 \\ 0, & |x|>1 \end{cases}$ is a symmetrical finite step function.

Find the Fourier cosine transform of $f(x)$.(10%)

8. (a) Evaluate the integral $\int_{-\infty}^{\infty}\frac{\cos x}{x^2+a^2}dx$.(5%)

(b) Evaluate the integral $\int_{-\infty}^{\infty}\frac{x\sin x}{x^2+a^2}dx$.(5%)

9. An atom particle is confined inside a rectangular box of sides $a, b,$ and c. The particle is described by a wave function Ψ that satisfies the Schrodinger wave equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi=E\Psi.$$

The wave function is required to vanish at each surface of the box. The condition imposes costraints on the separation constants and therefore on the energy E. What is the smallest value of E for which such a solution can be obtained?(10%)

10. Green's function $G(\mathbf{r}_1,\mathbf{r}_2)$ is a solution of the equation $\nabla^2 G(\mathbf{r}_1,\mathbf{r}_2)=-\delta(\mathbf{r}_1-\mathbf{r}_2)$, where delta function $\delta(\mathbf{r}_1-\mathbf{r}_2)$ describes a point source at the point $\mathbf{r}_2$.

Evaluate the Green's function of three dimensional free space.(10%)