科目:微積分
考試時間:100 分鐘

系所:應用數學系
身份別:一般生、在職生　　　　是否使用計算機:否
本科原始成績:100 分

1. (10%) Determine whether the statement is True or False.

   (a) The graph of the parametric equations $x = t^2$ and $y = t^2$ is the line $y = x$.

   (b) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent.

   (c) Let $\mathbf{r}(t)$ be a vector function. If $|\mathbf{r}(t)| = 1$ for all $t \in \mathbb{R}$, then $|\mathbf{r}'(t)|$ is a constant.

   (d) If f is increasing and $f(x) > 0$ on an interval I, then $g(x) = 1/f(x)$ is decreasing on I.

   (e) If f' is continuous on $[1, 3]$, then $\int_1^3 f'(x)dx = f(3) - f(1)$.

2. Find the limit.

   (a) (5%) $\displaystyle\lim_{x\to -2^+} (x+3)\frac{|x+2|}{x+2}$

   (b) (5%) $\displaystyle\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

   (c) (5%) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + \frac{3i}{n}\right)^3 \left(\frac{3}{n}\right)$

   (d) (5%) $\displaystyle\lim_{\theta\to 0} \frac{\cos\left(\frac{\pi}{2} + \theta\right)}{\theta}$

3. Find the derivative.

   (a) (5%) $f(x) = (\sin x)^{\ln x}$

   (b) (5%) $g(x) = \displaystyle\int_1^{e^x} \frac{2\ln t}{t} dt$

4. Evaluate the integral.

   (a) (10%) $\displaystyle\int \frac{x^2 - 2x - 1}{(x-1)^2(x^2+1)} dx$

   (b) (10%) $\displaystyle\int \sin\sqrt{x}\, dx$

5. (10%) Show that the function

$$f(x, y) = \frac{x^2 y}{x^4 + y^2}$$

has no limit as $(x, y) \to (0, 0)$.

6. (10%) Show that the function

$$g(x) = \begin{cases} x^2 \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is differentiable at 0.

國立高雄大學 104 學年度研究所碩士班招生考試試題

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7. (10%) Find the equations of the tangent plane and the normal line to $x^3y - y^2 + z^2 = 7$ at the point $(1, 2, 3)$.

8. (10%) Evaluate

$$\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2} \, dy \, dx.$$