招生學年度	104	招生類別	碩士班
系所班別	應用數學系碩士班		
科目名稱	線性代數		
注意事項	本考科禁止使用掌上型計算機		

1. (10%) Discover whether

$$A=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

is invertible, and find A^{-1} if it exists.

2. (10%) Let

$$A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ -1 & 0 & 3 & 5 \\ 1 & -2 & 1 & 1 \end{pmatrix}.$$

Find a row-reduced echelon matrix R which is row-equivalent to A and an invertible 3×3 matrix P such that $R=PA$.

3. (10%) Discover whether

$$\mathsf{W}=\left\{(a_1,a_2,a_3,a_4,a_5)\in\mathbb{R}^5 : a_1-2a_2+3a_3-a_4+2a_5=0\right\}$$

is a subspace of $\mathbb{R}^5$, and find its dimension if it is a subspace. Justify your answers.

4. (12%) If W_1 and W_2 are finite-dimensional subspaces of a vector space V (over $\mathbb{R}$), then prove that $\mathsf{W}_1+\mathsf{W}_2$ is finite-dimensional and

$$\dim(\mathsf{W}_1)+\dim(\mathsf{W}_2)=\dim(\mathsf{W}_1\cap\mathsf{W}_2)+\dim(\mathsf{W}_1+\mathsf{W}_2).$$

5. (8%) Let

$$\mathsf{W}_1=\left\{\begin{pmatrix} a & b \\ c & a \end{pmatrix} : a,b,c\in\mathbb{R}\right\} \text{ and } \mathsf{W}_2=\left\{\begin{pmatrix} 0 & d \\ -d & e \end{pmatrix} : d,e\in\mathbb{R}\right\}$$

be subspaces of $\mathsf{M}_{2\times 2}(\mathbb{R})$. Is $\mathsf{M}_{2\times 2}(\mathbb{R})=\mathsf{W}_1+\mathsf{W}_2$? Justify your answer.

6. (10%) Let

$$A=\begin{pmatrix} 1 & 0 & -1 & 2 & 1 \\ -1 & 1 & 3 & -1 & 0 \\ -2 & 1 & 4 & -1 & 3 \\ 3 & -1 & -5 & 1 & -6 \end{pmatrix}.$$

Find a 5×5 matrix M with rank 2 such that $AM=O$, where O is the 4×5 zero matrix.

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7. (10%) Let T be the linear operator on $\mathbb{R}^4$, defined by

$$\mathsf{T}(a, b, c, d) = (a + 3b, 4a + 2b, c + d, 4c + d).$$

Determine whether T is diagonalizable and if it is, find an ordered basis β for $\mathbb{R}^4$ such that $[\mathsf{T}]_\beta$ is a diagonal matrix.

8. (10%) Let V be an inner product space (over $\mathbb{R}$). Prove the Cauchy-Schwarz inequality.

9. (10%) Given the subspace $\mathsf{W} = \{(x, y, z) \in \mathbb{R}^3 : x - 2y + z = 0\}$ and $x = (1, 1, 2)$, find the orthogonal projection of x on W.

10. (10%) Let V be a finite-dimensional inner product space (over $\mathbb{R}$), and let $g : \mathsf{V} \to \mathbb{R}$ be a linear transformation. Prove that there exists a unique vector $y \in \mathsf{V}$ such that $g(x) = \langle x, y \rangle$ for all $x \in \mathsf{V}$, where $\langle x, y \rangle$ denotes the inner product of x and y.