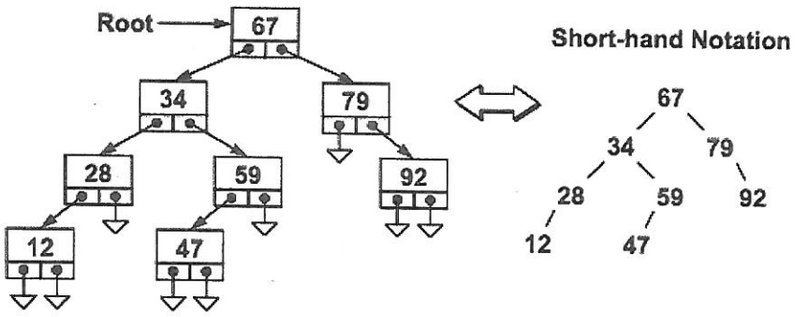
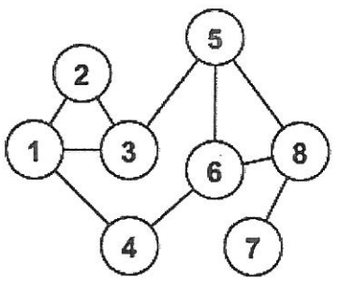
Briefly answer the following question (20% for each).

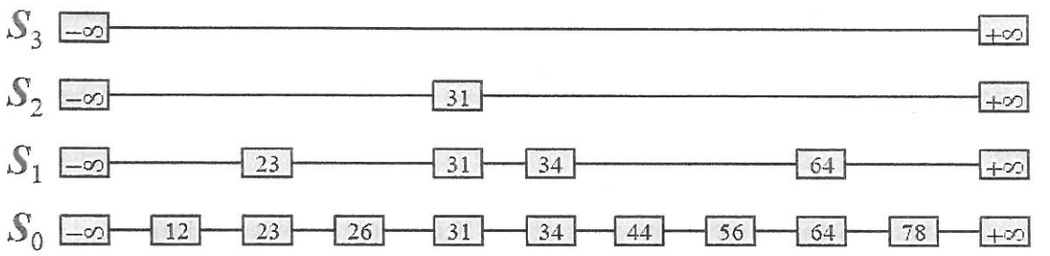
1. Use the following binary tree to answer the questions: (a). please list the traveling order in the pre-order and post-order; (b). Please use the heap sort method one-by-one to remove the key 67, and 24. And then list the two trees after removing key 67 and key 24; (c) Please write the in-order traveling algorithm for the binary tree.



2. Use the following graph to answer the question. (a). Please draw the adjacent list and matrix for the graph; (b) Based on your adjacent list (note: not adjacent matrix), please list travelling nodes to construct a spanning tree, if its starting node is node 6, in the *depth-first* and width-first orders, respectively; (c) Please write the minimal spanning tree algorithm (either depth-first or width- first) for a undirected graph.



3. For the following Skip-list, (a) please redraw the skip list again in your answer sheet and mark the traversing path to find the key 43 (not found); (b) assume we need to insert two records with their keys being 75 and 14. Assume after the insertion for key 75 in list S0, the next three results of flipping an coin about whether to insert an index in S1, S2 and S3 are head, flower, and head; and after the insertion for key 14 in list S0, the next three results of flipping an coin about whether to insert an index in S1, S2 and S3 are head, head, and flower. If the result of flipping a coin is head, the concerned list will be inserted an index; otherwise, will not. Please redraw the skip list after inserting the two keys according to the results of the flipping a coin; (c) please write the insertion algorithm for the insertion.



4. (a) Write the prefix form of the following expressions:
   (1). a && -6 || c-d*g || !(e>f) && a+b
   (2). !(a && !((b<= c) || (c =>d))) || (c < (-e*-b))
   (b) If an expression tree whose preorder, inorder and postorder traversals are +*AB+*CDE, A*B+C*D+E, and AB*CD*E++, respectively. Please draw the expression tree;
   (c) for a binary tree, *if n0 is the number of leaf nodes and n2 the number of nodes of degree 2, please prove n0=n2+1.*

5. Suppose we start with n sets, each containing a distinct element. (a) please prove that if k unions are performed, then no set contains more than k+1 elements; (b) prove that if fewer than $\lceil n/2 \rceil$ unions are performed, then at least one set with a single element in it remains; (c) prove that if u unions are performed, then at least max{n-2u, 0} singleton sets remain.