1. (10 %) Determine and sketch the convolution of the two signals in figure 1.

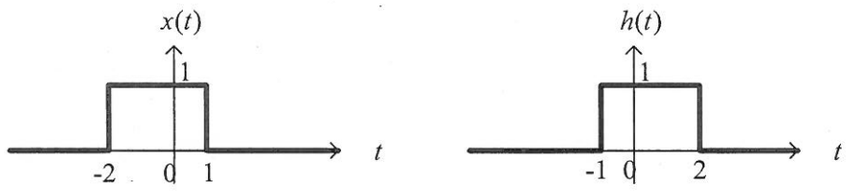

Figure 1.

2. (20%) Consider three systems with the following input-output relationships:

System 1: $y[n] = \begin{cases} x\left[\frac{n}{4}\right], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$,

System 2: $y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$,

System 3: $y[n] = x[5n]$,

Suppose that these systems are connected in series. Find the input-output relationship for the overall interconnected system. Is this system linear?

3. (15%) For the continuous-time periodic signal

$$x(t) = 1 + 2\cos(\frac{\pi}{2}t) + 4\sin(\frac{\pi}{3}t)$$

determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

4. (20%) A causal and stable LTI system S has the frequency response

$$H(j\omega) = \frac{j\omega + 1}{15 - \omega^2 + 8j\omega}$$

(a) (10%) Determine a differential equation relating the input $x(t)$ and output $y(t)$ of S.

(b) (5%) Determine the impulse response $h(t)$ of S.

(c) (5%) What is the output of S when the input is

$$x(t) = e^{-t}u(t) + 2te^{-t}u(t)$$

5. (20%) Consider a signal $x[n]$ which is the product of two other signals; that is,

$$x[n] = x_1[n]x_2[n], \text{ where}$$

$$x_1[n] = \frac{\sin(\frac{3\pi n}{4})}{\pi n} \text{ and } x_2[n] = \frac{\sin(\frac{\pi n}{2})}{\pi n}$$

(a) (10%) Determine and sketch the frequency response of $x_1[n]$.

(b) (10%) Determine and sketch the frequency response of $x[n]$.

6. (15%) We have a disc rotating clockwise at a constant rate ω_0 with a single radial line marked on the disc, as shown in Figure 2. The flashing strobe acts as sampling system with frequency ω_s. What are the observed rotating frequency and the rotating direction of the disc for three cases as below?

(a) (5%) $\omega_s = 4\omega_0$.

(b) (5%) $\omega_s = \frac{7}{4}\omega_0$.

(c) (5%) $\omega_s = \frac{1}{3}\omega_0$.

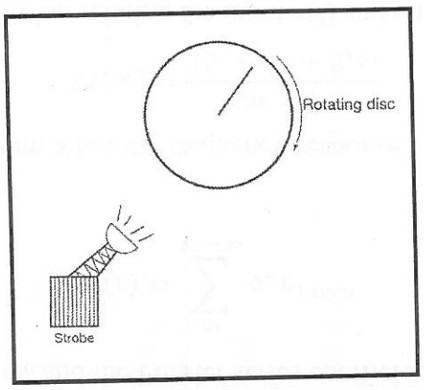

Figure 2.